# TEL-AVIV UNIVERSITY 

THE IBY AND ALADAR FLEISCHMAN FACULTY OF ENGINEERING

# Heat Transfer Design in Thermoelectric System Based on Vapor Chambers 

By
Eduard Itschakov, 308624014

June 2015

This work was carried out under the supervision of Dr. Alexander Gurevich


#### Abstract

Thermoelectric cooling (TEC) creates a heat flux between the junctions of two different types of materials. TEC transfers heat from one side of the device to the other, with consumption of electrical energy, depending on the direction of the current.

An extensive use of thermoelectric technology in various applications from space industry to the field of refrigerated food transportation holds a number of important advantages: accurate temperature control and an ability to cool small areas of a component up to the level of the single electronic component.

Dimensions of TEC range from a few millimeters up to $60-70 \mathrm{~mm}$, while a heat exchanger dimensions can reach up to the hundreds of millimeters. Because a TEC component covers a small area of a heat exchanger, efficiency of a thermoelectric system decreases and results in low heat dissipation. One of the options to solve the heat dissipation problem is to use the Heat Pipe technology that allows heat transfer with low temperature differentials. A recent development of the Heat Pipe technology has led to a design of a new Heat Pipe type that is a Flat Heat Pipe called Vapor Chamber. The use of Vapor Chamber technology allows homogeneous temperature dissipation over the surfaces of a hot or cold source. This technology allows integrating between TEC components and Vapor Chambers for building up effective thermoelectric systems.

Currently, there is insufficient scientific information on the performance of TEC components mounted on Vapor Chambers. As an integration of these components is relatively new, there is a lack of information on the effective conductivity of an integrated thermoelectric system.

Within the framework of this project, I will study the heat dissipation capacity of the system that consists of a TEC component and a Vapor Chamber component made of aluminum material. I will calculate thermal conductivity of Vapor Chamber as a function of TEC and Vapor Chamber dimensions that will allow designing the most efficient thermoelectric systems in future.


As part of the project, I will perform several experiments on an integrated thermoelectric system. I will examine the impact of the components relative location on efficient thermal conductivity of Vapor Chamber.

In addition, I will perform thermal calculations and heat transfer analysis of the integrated thermoelectric system to obtain effective conductivity.

Also, this project will include the review of selected scientific works dealing with thermoelectric systems, thermal and mechanical design, a series of thermal experiments and thermal analyses.

## The project stages are as follows:

- Literature review
- Experimental study of temperature distribution over the surface of Vapor Chamber at different dimensions and power dissipations of TEC mounted on Vapor Chamber
- Preparing a thermal model and performing a thermal analysis of the Vapor Chamber TEC system dimensions and power dissipations used in experimental study
- Calculating an efficient thermal conductivity of the Vapor Chamber as a function of dimensions and power dissipations
- Conclusions


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## 1. Introduction

### 1.1. Thermoelectric Technology Overview

Thermoelectric technology has become the method of choice for fast, precise and compact temperature control. The technology is based on two physical effects:

- The Seebeck effect
- The Peltier effect

The Seebeck effect was discovered by Thomas Johann Seebeck in 1821. He has accidentally found a voltage existed between two ends of a metal bar when a temperature gradient existed within the bar.

A temperature difference causes diffusion of electrons from the hot side to the cold side of a conductor. The motion of electrons creates voltage between two ends of the bar. The voltage is proportional to the temperature difference as governed by: $\mathrm{V}=\alpha(\mathrm{Th}-\mathrm{Tc})$, where $\alpha$ is the Seebeck coefficient of the bar's material.


Figure 1: Seebeck Effect - Voltage Generated by Temperature Differences

The Peltier effect is named after Jean Charles Peltier (1785-1845) who first observed it in 1834. To describe Peltier effect In Peltier effect devices, a temperature difference is created: one
junction becomes cooler and one junction becomes hotter. Although Peltier coolers are not as efficient as some other types of cooling devices, they are accurate, easy to control.


Figure 2: Peltier Effect - Heating or Cooling Process Depending on the Current Flow Direction
Construction: Two unique semiconductors, one n-type and one p-type, are used because they need to have different electron densities. The semiconductors are placed thermally in parallel to each other and electrically in series and then joined with a thermally conducting plate on each side. When a voltage is applied to the free ends of the two semiconductors there is a flow of DC current across the junction of the semiconductors causing a temperature difference. The side with the cooling plate absorbs heat, which is then moved to the other side of the device where the heat sink is. The semiconductor elements are typically connected side by side and sandwiched between two ceramic plates. The cooling ability of the total unit is then proportional to the number of elements in it.


Figure 3: Schematic of Thermoelectric Cooler
Individual couples are connected electrically in series and thermally in parallel. Couples are thermally connected by a ceramic that has high electrical resistivity and high thermal conductivity.


Figure 4: Thermal Flow

### 1.2. Heating and Cooling Process

Thermoelectric cooler (TEC) is a heat pump, absorbing heat from one side and dissipating heat from another. A thermoelectric cooler provides a temperature gradient(around $70^{\circ} \mathrm{C}$ ) between the ceramic plates. Because of temperature gradients existing on the thermal interfaces on the both sides of TEC an efficient $\Delta \mathrm{T}$ between the cold and hot media is less than the maximum $\Delta \mathrm{T}$ that the thermoelectric material can inherently create. The significant thermal resistances are created between the two interfaces: the side of heat absorption from cooling object and the side of heat dissipation to heat sink.

Efforts to minimize these thermal resistances between the interfaces have played a big role in the development history of thermoelectric coolers.

In the interfaces, there are copper electrode, ceramics, grease and air. The thermal conductivities of these interface materials are shown in Table 1. Among materials shown in Table 1, we can see values of a significantly low thermal conductivity (and a significantly high thermal resistivity) of thermo-element, by which we can get high temperature difference in the thermoelectric cooler. Using the Table 1, we can also find an extremely low thermal air conductivity (and high thermal air resistivity). We use grease to fill the air gap between the interfaces. But, a grease is usually made of organic substances, so thermal resistivity of greases range from 10 times ( ZnO or alumina powder added) to 100 times (grease without good thermal conductivity additive like vacuum grease) of that of alumina.

Roughly talking, power on heat dissipation side is two -three times larger than that of heat absorption side. So, effort should be made to minimize the thermal resistance of the heat dissipation side.

Table 1: Thermal conductivity and thermal resistivity of different materials

| Substance/Material | Thermal Conductivity <br> $\left(\mathbf{W} / \mathbf{m}^{\circ} \mathbf{C}\right)$ | Thermal resistivity (m$\left.{ }^{\circ} \mathbf{C} / \mathbf{W}\right)$ |
| :--- | :---: | :---: |
| TEC element | 1.5 | 0.67 |
| Copper | 400 | 0.0025 |
| $\mathbf{9 6 \%}$ alumina | 20 | 0.05 |
| Aluminum Nitride | 170 | 0.006 |
| Greases | $0.2 \sim 2.0$ | $0.5 \sim 5$ |
| Air | 0.03 | 33 |



Figure 5: TEC and Heat Sink System
As alumina had the best cost/performance as the pad of TECs, it has become the standard pad material of today's TECs. Aluminum nitride (AlN) has become a candidate pad material recently, but it is too expensive to be widely used. However, AlN is yet an attractive material with its 10 -fold superiority of thermal conductivity than alumina.

The Table 2 below shows useful materials and substances for reducing spreading thermal resistance. Among them, Heat Pipes, the component having different heat conduction physics. The Heat Pipes do not utilize solid heat conduction but utilize latent heat of liquid vapor. Their near sound velocity to move has thermal conductivity that is 10 times larger than diamond or 100 times larger than copper. This performance is far beyond other heat conductive materials. Heat Pipes are widely used owing to its special characteristics - an ability to dissipate heat far from the heat source. However, when you use a plate-like Heat Pipe to spread a TEC power over surface of heat dissipating elements, it has a great effect in Reducing temperature differential between TEC and heat dissipating elements.

Materials having high heat conduction performance both for horizontal/vertical directions are diamond and cubic boron nitride (CBN). The material having potential to be used as the pads of TECs may be diamond-like carbon (DLC), if we will be able to economically make a thick DLC plate in future. DLC has far better thermal conductivity than copper, and the source materials are cheap (benzene or methane gases).
The Table 2 shows the materials that have low thermal properties compared to the Heat Pipe.
Table 2: Good Thermal-Conductive Material

| Substance/Material | Thermal Conductivity <br> $\left(\mathbf{W} / \mathbf{m}^{\circ} \mathbf{C}\right)$ | Thermal resistivity (m <br> W $\mathbf{C} /$ <br> $\mathbf{W})$ |
| :--- | :---: | :---: |
| Diamond | 2000 | $5 \times 10^{-4}$ |
| DLC (diamond like <br> carbon) | 700 | $1.4 \times 10^{-3}$ |
| CBN (cubic boron <br> nitride) | 1300 | $7.7 \times 10^{-4}$ |
| Heat Pipe | $\mathbf{3 0 0 0 0}$ | $\mathbf{3 . 3 \times 1 0 ^ { - 5 }}$ |

### 1.3. Heat Pipe Technology Overview

Heat Pipes transfer heat more efficiently and evenly than solid conductors, such as aluminum or copper, because of their lower total thermal resistance. The Heat Pipe is a hermetically sealed device with high vacuum filled with a small quantity of working fluid (water, acetone, nitrogen, methanol, ammonia, or sodium). Heat is absorbed by vaporizing the working fluid. The vapor transports heat to the condenser region where the condensed vapor releases heat to a cooling medium. The condensed working fluid is returned to the evaporator by gravity, or by the Heat Pipe's wick structure, creating capillary action. Both cylindrical and planar Heat Pipe variants have an inner surface lined with a capillary wicking material.


Figure 6: Schematic of Heat Pipe Structure
Heat Pipe is the most common passive, capillary-driven two-phase system. Two-phase heat transfer involves the liquid-vapor phase change (boiling/evaporation and condensation) of a working fluid. Heat Pipes have an extremely effective high thermal conductivity. While solid conductors such as aluminum, copper, graphite and diamond have thermal conductivities ranging from $250(\mathrm{~W} / \mathrm{m} \cdot \mathrm{K})$ to $1,500(\mathrm{~W} / \mathrm{m} \cdot \mathrm{K})$, Heat Pipes have effective thermal conductivities that range from $5,000(\mathrm{~W} / \mathrm{m} \cdot \mathrm{K})$ to $200,000(\mathrm{~W} / \mathrm{m} \cdot \mathrm{K})$. Heat Pipes transfer heat from the heat source (evaporator) to the heat sink (condenser) over relatively long distances through the latent heat of vaporization of a working fluid. Heat Pipes typically have 3 sections: an evaporator section (heat input/source), adiabatic (or transport) section and a condenser section (heat output/sink).

The three major components of a Heat Pipe include:

- A vacuum tight, sealed containment shell or vessel
- Working fluid
- Capillary wick structure

They all work together to transfer heat more efficiently and evenly. The wick structure lines the inner surface of the Heat Pipe shell and is saturated with the working fluid. The wick provides the structure to develop the capillary action for the liquid returning from the condenser (heat output/sink) to the evaporator (heat input/source). Since the Heat Pipe contains a vacuum, the working fluid will boil and take up latent heat at well below its boiling point at atmospheric pressure. Water, for instance, will boil at just above $273^{\circ} \mathrm{K}\left(0^{\circ} \mathrm{C}\right)$ and start to effectively transfer latent heat at this low temperature.

### 1.4. Vapor Chamber Technology Overview

The principle of operation for Vapor Chamber is similar to that of Heat Pipes. Both are heat spreading devices with highly effective thermal conductivity due to phase change phenomena. A Vapor Chamber is a basically a flat Heat Pipe that can be part of the base of a heat sink.

A Vapor Chamber is a vacuum vessel with a wick or micro channel structure lining its inside walls. The wick is saturated with a working fluid. The choice of this fluid is based on the operating temperature of the application and compatibility with shell material.

As heat is applied, the fluid at that location immediately vaporizes and the vapor rushes to fill the vacuum. Wherever the vapor comes into contact with a cooler wall surface, it condenses, releasing its latent heat of vaporization. The condensed fluid returns to the heat source via capillary action at the wick, ready to be vaporized again and repeat the cycle.

The capillary action of the wick enables the Vapor Chamber to work in any orientation, though its performance is orientation dependent. The pressure drop in the vapor and the liquid determines the capillary limit or the maximum heat carrying capacity of the Heat Pipe.

A Vapor Chamber is different from a Heat Pipe in that the condenser covers the entire top surface of the structure. In Vapor Chamber, heat transfer in two directions and is planar. In a Heat Pipe, heat transmission is in one direction and linear. In the two-phase Vapor Chamber device, the rates of evaporation, condensation, and fluid transport are determined by the Vapor Chambers geometry and the wicks structural properties.


Figure 7: Schematic of Vapor Chamber

### 1.5. Types of Vapor Chambers and Production Methods

The following two Vapor Chamber types are usually used: the Vapor Chamber with internal channels and the Vapor Chamber composed of several parts. Both Vapor Chamber types manufacturing include the sealing part that uses the brazing method. This method implies the following operation: in a metal-joining process, a filler metal is heated above melting point and distributed between two or more close-fitting parts by capillary action. The filler metal is brought slightly above its melting (liquids) temperature while protected by a suitable atmosphere, usually a flux. It then flows over the base metal (known as wetting) and is then cooled to join the work pieces together. It is similar to soldering, except the temperatures used to melt the filler metal are higher for brazing.

## Type 1 - Vapor Chamber with Internal Channels

A Vapor Chamber includes integrally formed main body defining a chamber with channels. The channels are outwardly extended from one side of the main body in a direction opposite to the chamber. The Vapor Chamber is a vacuum container with wick structure lining the inside walls that is saturated with working fluid.

The method of manufacturing a Vapor Chamber of this type includes:

- Using an extrusion process to manufacture a main body having a plurality channels
- Sealing two ends of the main body using the Brazing method
- Filling the chamber with a working fluid

This method of manufacturing the Vapor Chamber structure allows using reduced material and labor costs, and shortens manufacturing time.


Figure 8: Vapor Chamber with Internal Channels

## Type 2 - Vapor Chamber Composed from Several Parts - Copper Material

A Vapor Chamber of this type is composed of several parts, usually of the main body and the cover . The two parts are sealed together using the Brazing method. To disperse working fluid throughout the inner area, a porous material is used. The porous material transfers back the liquid to the hot areas.


Figure 9: Vapor Chamber Composed of Several Parts

When manufacturing both types of Vapor Chambers, it is very important to keep an internal vacuum. Otherwise, the outside air may enter inside and cause a pressure rise. The pressure rise will affect the temperature rise and ,consequently, the thermodynamic phase change will be different.

Applying any of the above Vapor Chamber types characterized by a particular operating temperature range. Therefore, the design of the Vapor Chamber must account for the intended temperature range by specifying the proper working fluid.

Longevity of a Vapor Chamber can be assured by selecting a container, a wick and welding materials that are compatible with one another and with the working fluid of interest.

Performance can be degraded and failures can occur in the container wall if any of the parts (including the working fluid) are not compatible. For instance, the parts can react chemically or set up a galvanic cell within the Vapor Chamber. Additionally, the container material may be soluble in the working fluid or may catalyze the decomposition of the working fluid at the expected operating temperature. A compilation of the most up-to-date information concerning the compatibility of metals with working fluids for Vapor Chamber is shown in Table 3.

Table 3: Materials Compatible and Incompatible to Working Fluid

| Working <br> Fluid | Compatible Material | Incompatible Material |
| :--- | :--- | :--- |
| Water | Copper, Silica, Nickel, Titanium | Aluminum, Inconel |
| Ammonia | Aluminum, Stainless Steel, Cold <br> Rolled Steel, Iron, Nickel |  |
| Methanol | Stainless Steel, Iron, Copper, Brass, <br> Silica | Aluminum |
| Acetone | Aluminum, Stainless Steel, Copper, <br> Brass, Silica |  |
| Freon-11 | Aluminum |  |
| Freon-12 | Aluminum, Iron |  |


| Freon-113 | Aluminum |  |
| :--- | :--- | :--- |
| Heptane | Aluminum |  |
| Dowtherm | Stainless Steel, Copper, Brass | Tungsten, Tantalum, Molybdenum, <br> Niobium |
| Lithium | Stainless Steel, Nickel, Inconel, <br> Titanium |  |
| Sodium | Stainless Steel, Nickel, Inconel, <br> Niobium | Titanium |
| Cesium | Titanium, Niobium, Stainless Steel, <br> Nickel based super alloys |  |
| Mercury | Stainless Steel | Molybdenum, Nickel, Tantalum, <br> Inconel, Titanium, Niobium |
| Lead | Tungsten, Tantalum | Stainless Steel, Nickel, Inconel, <br> Titanium, Niobium |
| Silver | Tungsten, Tantalum | Rhenium |

### 1.6. Project Scope

Within the framework of this project, I use the Extruded Vapor Chamber type made of aluminum to find its effective conductivity characteristics. For this purpose, I will compare results received from the SolidWorks thermal simulation tool and the laboratory thermal experiment findings.

I have selected the Extruded Vapor Chamber type because of its benefits:

- High reliability
- Simplicity of manufacture
- Low weight
- High thermal conductivity


## 2. Thermal Calculations

### 2.1.Calculation of Forced Convection Heat Transfer Coefficient

The Figure 10 below illustrates the assembly including a Vapor Chamber with fins and TEC.


Figure 10: Mechanical Characteristics
The Figure 11 presents mechanical dimensions of each of the components. TEC position will vary in each experiment.


Figure 11: Mechanical Dimensions
To set thermal environmental and boundary conditions to SolidWorks Thermal Simulation tool, it is required to find a Forced Convection Heat Transfer Coefficient $h$. For this purpose, I will take one of the fin channels and assume that this channel serves as a tube.

## Setting a flow type:

Based on the data displayed in Table 4, air properties for the $25^{\circ} \mathrm{C}$ temperature are as follows:
$\mu=1.8 \times 10^{-5}\left[\frac{\mathrm{~kg}}{\mathrm{~m} \times \mathrm{sec}}\right]-$ Air Dynamic Viscosity
$\rho=1.205\left[\frac{\mathrm{~kg}}{\mathrm{~m}^{3}}\right]-$ Air Density
$\mathrm{Cp}=1005\left[\frac{\mathrm{~J}}{\mathrm{~kg} \times{ }^{\circ} \mathrm{K}}\right]-$ Specific Heat
$\mathrm{k}=0.0257\left[\frac{\mathrm{~W}}{\mathrm{~m} \times{ }^{\circ} \mathrm{K}}\right]-$ Thermal Conductivity
$\operatorname{Pr}=0.713-$ Prandtl's Number

Table 4: Air Properties for Atmospheric Pressure

| $\frac{\text { Temperature }}{-t-} \begin{aligned} & \left.\rho^{\circ} \mathrm{C}\right) \end{aligned}$ | $\begin{aligned} & \frac{\text { Density }}{-\rho-} \\ & \left(\mathrm{kg} / \mathrm{m}^{3}\right) \end{aligned}$ | $\begin{aligned} & \text { Specific Heat } \\ & -c_{p} . \\ & (k J /(k g K)) \end{aligned}$ | Thermal Conductivity - $k$ ( $W /(m K)$ ) | $\begin{aligned} & \frac{\text { Kinematic }}{\text { Viscosity }} \\ & \times 10^{-6}\left(\mathrm{~m}^{2} / \mathrm{s}\right) \end{aligned}$ | Expansion Coefficient $\begin{gathered} -b- \\ \times 10^{-3}(1 / K) \end{gathered}$ | Prandtl's Number $-P_{r}-$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| -150 | 2.793 | 1.026 | 0.0116 | 3.08 | 8.21 | 0.76 |
| -100 | 1.980 | 1.009 | 0.0160 | 5.95 | 5.82 | 0.74 |
| -50 | 1.534 | 1.005 | 0.0204 | 9.55 | 4.51 | 0.725 |
| 0 | 1.293 | 1.005 | 0.0243 | 13.30 | 3.67 | 0.715 |
| 20 | 1.205 | 1.005 | 0.0257 | 15.11 | 3.43 | 0.713 |
| 40 | 1.127 | 1.005 | 0.0271 | 16.97 | 3.20 | 0.711 |
| 60 | 1.067 | 1.009 | 0.0285 | 18.90 | 3.00 | 0.709 |
| 80 | 1.000 | 1.009 | 0.0299 | 20.94 | 2.83 | 0.708 |
| 100 | 0.946 | 1.009 | 0.0314 | 23.06 | 2.68 | 0.703 |
| 120 | 0.898 | 1.013 | 0.0328 | 25.23 | 2.55 | 0.70 |
| 140 | 0.854 | 1.013 | 0.0343 | 27.55 | 2.43 | 0.695 |
| 160 | 0.815 | 1.017 | 0.0358 | 29.85 | 2.32 | 0.69 |
| 180 | 0.779 | 1.022 | 0.0372 | 32.29 | 2.21 | 0.69 |
| 200 | 0.746 | 1.026 | 0.0386 | 34.63 | 2.11 | 0.685 |
| 250 | 0.675 | 1.034 | 0.0421 | 41.17 | 1.91 | 0.68 |
| 300 | 0.616 | 1.047 | 0.0454 | 47.85 | 1.75 | 0.68 |
| 350 | 0.566 | 1.055 | 0.0485 | 55.05 | 1.61 | 0.68 |
| 400 | 0.524 | 1.068 | 0.0515 | 62.53 | 1.49 | 0.68 |

The fan properties are as follows, where the Fan type is NMB 2410ML-05W-B80, Axial:


Figure 12: Fan Curve
Rotor speed: $555.015 \mathrm{rad} / \mathrm{s}$
Outer diameter: 0.060 m
Hub diameter: 0.036 m
Direction of rotation: Counter - clockwise
The fin channel dimensions are as follows (see Figure 11):

Height $b=60[\mathrm{~mm}]$
Width $\mathrm{a}=2.3[\mathrm{~mm}]$,
$\mathrm{n}=21$ number of channels per each fan.
Hydraulic Diameter :

$$
\mathrm{Dh}=\frac{4 \times \text { area }}{\text { perimeter }}=\frac{4 \times \mathrm{a} \times \mathrm{b}}{2 \times(\mathrm{a}+\mathrm{b})}=\frac{2 \times \mathrm{a} \times \mathrm{b}}{(\mathrm{a}+\mathrm{b})}=\frac{2 \times 2.3 \times 60}{2.3+60}=4.43[\mathrm{~mm}]=4.43 \times 10^{-3}[\mathrm{~m}]
$$

The maximum fan volumetric flow rate is:

$$
\mathrm{Q}=1.05\left[\mathrm{~m}^{3} / \mathrm{min}\right]=\frac{1.05}{60}=0.0175\left[\mathrm{~m}^{3} / \mathrm{sec}\right] .
$$

Flow velocity is calculated as follows: $\mathrm{v}=\frac{\mathrm{Q}}{\mathrm{n} \times \mathrm{axb}}=\frac{0.0175}{21 \times 60 \times 2.3 \times 10^{-6}}=6.04[\mathrm{~m} / \mathrm{sec}]$
Reynolds Number calculation:

$$
\operatorname{Re}=\frac{\mathrm{v} \times \operatorname{Dh} \times \rho}{\mu}=\frac{6.04 \times 4.43 \times 10^{-3} \times 1.205}{1.8 \times 10^{-5}}=1791.2
$$

The calculated Reynolds Number is lower than 2500, consequently, the flow we have is of a laminar type (for maximum fan volumetric flow rate).

## Defining the operating point of fan curve:

Flow static pressure equation is:

$$
\begin{aligned}
& \Delta \mathrm{P}=4 \times \mathrm{f} \times \frac{\mathrm{L}}{\mathrm{D}_{\mathrm{h}}} \times \frac{\rho \times \mathrm{v}^{2}}{2 \times \mathrm{g}}=4 \times \frac{24 \mathrm{x} \mathrm{\mu}}{\mathrm{v} \times \mathrm{Dh} \times \rho} \times \frac{\mathrm{L}}{\mathrm{D}_{\mathrm{h}}} \times \frac{\rho \times \mathrm{v}^{2}}{2 \times \mathrm{g}}=\frac{48 \times \mu \times \mathrm{L} \times \mathrm{v}}{\mathrm{Dh}^{2} \times \mathrm{g}} \\
& \Delta \mathrm{P}=\frac{48 \times \mu \times \mathrm{L}}{\mathrm{Dh}^{2} \times \mathrm{g}} \times \frac{\mathrm{Q}}{\mathrm{n} \times \mathrm{a} \times \mathrm{b}}=\frac{48 \times \mu \times \mathrm{L} \times \mathrm{Q}}{\mathrm{Dh}^{2} \times \mathrm{g} \times \mathrm{n} \times \mathrm{a} \times \mathrm{b}} \\
& \Delta \mathrm{P}=\frac{96 \times 1.8 \times 10^{-5} \times 0.37 \times \mathrm{Q}}{21 \times\left(4.43 \times 10^{-3}\right)^{2} \times 9.86 \times 60 \times 2.3 \times 10^{-6}}=4280.28 \mathrm{xQ}
\end{aligned}
$$

Where:
$f$ - Flow Friction Factor is for the laminar flow according to Figure 13:

### 6.25 Fin Efficiency Factor

Table 6.7 Laminar Flow Friction Factors ( $f$ ) for Fully Developed Velocity and Temperature Profiles in Smooth Pipès with Different Cross Sections.

| Geometry | Friction Factor |
| :---: | :---: |
|  | $f=\frac{13.3}{N_{R}}$ |
| $b \square \frac{b}{a}=1$ | $f=\frac{14.2}{N_{R}}$ |
| $\bigcirc$ | $f=\frac{18}{N_{R}}$ |
|  | $f=\frac{18.3}{N_{R}}$ |
| $a \square \frac{b}{a}=8$ | $f=\frac{20.5}{N_{R}}$ |
| $\frac{b}{a}=\infty$ | ${ }^{\prime} f=\frac{24}{N_{R}}$ |

Figure 13: Friction Factor
$\frac{\mathrm{a}}{\mathrm{b}}=\frac{60}{2.3}=26.08>8 \Rightarrow \mathrm{f}=\frac{24}{\operatorname{Re}}=\frac{24 \mathrm{x} \mu}{\mathrm{v} \times \operatorname{Dh} \times \rho}$
$\mathrm{L}=320[\mathrm{~mm}]=0.32[\mathrm{~m}]$ - fin length
$\mathrm{v}=\frac{\mathrm{Q}}{\mathrm{a} \times \mathrm{b}}=\frac{4 \times \mathrm{Q}}{\mathrm{axb}}-$ flow velocity
The operating point of fan curve is according to Table 5 and Figure 14:
$\Delta P=50[P a], Q=\frac{50}{4280.28}=0.0117\left[\mathrm{~m}^{3} / \mathrm{sec}\right]$

Table 5: Air Flow and Static Pressure

| Air Flow [m^3/sec] | Fan Static Pressure[Pa] | Flow Static Pressure [Pa] |
| :---: | :---: | :---: |
| 0 | 141 | 0 |
| 0.001666667 | 130 | 7.1338 |
| 0.003333333 | 126 | 14.2676 |
| 0.005 | 100 | 21.4014 |
| 0.006666667 | 87 | 28.5352 |
| 0.008333333 | 73 | 35.669 |
| 0.01 | 60.5 | 42.8028 |
| 0.011666667 | 50.5 | 49.9366 |
| 0.013333333 | 41 | 57.0704 |


| Air Flow [m^3/sec] | Fan Static Pressure[Pa] | Flow Static Pressure [Pa] |
| :---: | :---: | :---: |
| 0.015 | 30 | 64.2042 |
| 0.016666667 | 0 | 71.338 |



Figure 14: Fan and Flow Curves
The defined flow velocity is calculated as follows:

$$
\mathrm{v}=\frac{\mathrm{Q}}{\mathrm{n} \times \mathrm{axb}}=\frac{0.0117}{21 \times 60 \times 2.3 \times 10^{-6}}=4.04[\mathrm{~m} / \mathrm{sec}]
$$

New Reynolds Number is then calculated as follows:

$$
\operatorname{Re}=\frac{\mathrm{v} \times \mathrm{Dh} \times \rho}{\mu}=\frac{4.04 \times 4.43 \times 10^{-3} \times 1.205}{1.8 \times 10^{-5}}=1114
$$

## Calculating a Convection coefficient:

The convection coefficient value is calculated as follows:

$$
\mathrm{h}=\frac{\mathrm{Nu} \times \mathrm{k}}{\mathrm{Dh}}
$$

The Nusselts Number will be set using the Table 6 :

Table 6: Nusselt Number for Fully Developed Laminar Flow

| Tube Geometry | $\begin{gathered} a / b \\ \text { or } \theta^{\circ} \end{gathered}$ | Nusselt Number |  |
| :---: | :---: | :---: | :---: |
|  |  | $T_{s}=$ Const. | $\dot{q}_{s}=$ Const. |
| Circle | - | 3.66 | 4.36 |
| Rectangle | $\begin{aligned} & \hline \frac{a / b}{1} \\ & 2 \\ & 3 \\ & 4 \\ & 6 \\ & 8 \\ & \infty \end{aligned}$ | $\begin{aligned} & 2.98 \\ & 3.39 \\ & 3.96 \\ & 4.44 \\ & 5.14 \\ & 5.60 \\ & 7.54 \end{aligned}$ | $\begin{aligned} & 3.61 \\ & 4.12 \\ & 4.79 \\ & 5.33 \\ & 6.05 \\ & 6.49 \\ & 8.24 \end{aligned}$ |
|  | $\begin{array}{r} \frac{a / b}{1} \\ 2 \\ 4 \\ 8 \\ 16 \end{array}$ | $\begin{aligned} & 3.66 \\ & 3.74 \\ & 3.79 \\ & 3.72 \\ & 3.65 \end{aligned}$ | $\begin{aligned} & 4.36 \\ & 4.56 \\ & 4.88 \\ & 5.09 \\ & 5.18 \end{aligned}$ |
|  | $\theta$ <br> $10^{\circ}$ <br> $30^{\circ}$ <br> $60^{\circ}$ <br> $90^{\circ}$ <br> $120^{\circ}$ | $\begin{aligned} & 1.61 \\ & 2.26 \\ & 2.47 \\ & 2.34 \\ & 2.00 \end{aligned}$ | $\begin{aligned} & 2.45 \\ & 2.91 \\ & 3.11 \\ & 2.98 \\ & 2.68 \end{aligned}$ |

$\frac{\mathrm{b}}{\mathrm{a}}=\frac{60}{2.3}=26.08>8 \Rightarrow \mathrm{Nu}=7.54 \div 8.24=7.89$
In my calculation, (T) temperature and (q) heat flux are not constant, so I have selected a value for Nusselt Number as follows: $\mathrm{Nu}=\frac{7.54+8.24}{2}=7.89$

Given the above, the convection coefficient value is calculated as follows:
$\mathrm{h}=\frac{\mathrm{Nu} \times \mathrm{Ka}}{\mathrm{Dh}}=\frac{7.89 \times 0.0257}{4.43 \times 10^{-3}}=45.8\left[\frac{\mathrm{~W}}{\mathrm{~m}^{2} \times{ }^{\circ} \mathrm{C}}\right]$

### 2.2. Calculation of Fin Efficiency

Fin efficiency is defined as the ratio of heat that is actually transferred from the fin to the heat that would be transferred at the maximum uniform temperature distribution over the fins surface. The fin efficiency factor is:
$\eta=\frac{\tanh (m \times d)}{m \times d}=\frac{\tanh (33.12 \times 0.06)}{33.12 \times 0.06}=0.48=48 \%$

Where:
tanh - hyperbolic tangent
$\mathrm{m}=\sqrt{\frac{2 \times \mathrm{h}}{\mathrm{k} \times \delta}}=\sqrt{\frac{2 \times 45.8}{167 \times 0.0005}}=33.12$
$\mathrm{h}=45.8\left[\frac{\mathrm{~W}}{\mathrm{~m}^{2} \times^{\circ} \mathrm{C}}\right]$ - convection coefficient
$\mathrm{k}=167\left[\frac{\mathrm{~W}}{\mathrm{~m} \times{ }^{\circ} \mathrm{C}}\right]$ - fin thermal conductivity
$\delta=0.0005[\mathrm{~m}]$ - fin thickness
$\mathrm{d}=0.06[\mathrm{~m}]$ - effective height of fin from base
The calculated Fin Efficiency strengthens assume that T and q are not constant, so heat transfer conditions are between isothermal and adiabatic

### 2.3.Calculation of Temperature Differences

To calculate the temperature differences, I use the following equation:
$\dot{Q}=\mathrm{m} \times \mathrm{Cp} \times \Delta \mathrm{T} \Rightarrow \Delta \mathrm{T}=\frac{\dot{\mathrm{Q}}}{\mathrm{m} \times \mathrm{Cp}}$
$\Delta \mathrm{T} 1=\frac{57}{0.014 \times 1005}=4.05 \cong 4\left[{ }^{\circ} \mathrm{C}\right]$
$\Delta \mathrm{T} 2=\frac{114}{0.014 \times 1005}=8.1 \cong 8\left[{ }^{\circ} \mathrm{C}\right]$
Where:
$\dot{Q}$ - heat
$\mathrm{m}=\rho \times \mathrm{Q}=1.205 \times 0.0117=0.014[\mathrm{~kg} / \mathrm{sec}]-$ mass flow rate
$\mathrm{Cp}=1005\left[\mathrm{~J} / \mathrm{kg} \times{ }^{\circ} \mathrm{C}\right]-$ specific heat of substance
$\Delta \mathrm{T}=$ temperature differences

## 3. Solid Works Thermal Simulation

### 3.1. Displaying Solid Works Thermal Simulation Input Data

For setting $h$ (Convection Heat Transfer Coefficient) parameter I performed the Experiment 0 on an aluminum Fan box only. I checked this Experiment using the two Solid Works tools: Solid Works Thermal Simulation and Solid Works Flow Simulation (see Figure 15).


Figure 15: Experiment No. 0
Convection: lengths and fins were divided to equal areas, and for each area I applied the same convection coefficient and different temperatures (see Figure 16):


| P [w] | T11 ${ }^{\circ} \mathrm{Cl}$ | T2 $1^{\circ} \mathrm{Cl}$ | T3 ${ }^{\circ} \mathrm{Cl}$ | T4] ${ }^{\text {c }}$ | T5 ${ }^{\circ} \mathrm{C}$ | T6 $\left[^{\circ} \mathrm{Cl}\right.$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 57 | 22.5 | 23.5 | 24.5 | 25.5 | 26.5 | 27 |
| 110 | 23 | 25 | 27 | 29 | 30 | 31.5 |

Figure 16: Boundary Conditions for Experiment No. 0

Boundary conditions of Solid Works Flow Simulation for the Experiment No. 0 are shown in Table 7.

Table 7: Boundary Conditions

| Thermodynamic parameters | Static Pressure: $101325.00[\mathrm{~Pa}]$ <br> Ambient Temperature: see Figure 16 |
| :--- | :--- |
| Heat power | $57[\mathrm{w}]$ or $110[\mathrm{w}]$ |
| Solid parameters | Default material: Aluminum 6061,K=167 <br> $\left[\mathrm{W} / \mathrm{m}^{\circ} \mathrm{C}\right]$ |
| Initial Fluids | Air |
| Fan | Fan Type: NMB 2410ML-05W-B80- <br> Outlet flow vector direction: Normal to face <br> Environment pressure: 101325.00 Pa |
| Contact Resistances | Tflex220-0.02in(0.51mm)-100psi <br> Faces: Vapor Chamber and Fins Box |

The Experiment No. 0 results for 110 [w] are shown in Table 8 and Figure 17:
Table 8: Experiment No. 0 for 110[w] Results

| Thermal Simulation |  |  |  |
| :---: | :---: | :---: | :---: |
| $\mathrm{P}=110[\mathrm{w}]$ Tin=23${ }^{\circ} \mathrm{C}$ |  |  |  |
| Points | Experiment <br> Temperature | Simulation <br> Temperature | S.D. |
| 4 | 32 | 32.9 | 0.64 |
| 5 | 31.5 | 32.94 | 1.02 |
| 7 | 26.8 | 26.18 | 0.44 |
| 8 | 30 | 32.63 | 1.86 |


| Flow Simulation |  |  |  |
| :---: | :---: | :---: | :---: |
| $\mathrm{P}=110\left[\mathrm{w}\right.$ ] Tin= $\mathbf{2 3}^{\circ} \mathrm{C}$ |  |  |  |
| Points | Experiment <br> Temperature | Simulation <br> Temp | S.D. |
| 4 | 32 | 32.10 | 0.07 |
| 5 | 31.5 | 32.07 | 0.40 |
| 7 | 26.8 | 24.86 | 1.37 |
| 8 | 30 | 30.88 | 0.62 |




Figure 17: Experiment No.0, 110[w] Graphs

The experiment No. 0 results for 57[w] are shown in Table 9 and Figure 18.
Table 9: Experiment No. 0 Results for 57 [w]

| Thermal Simulation |  |  |  |
| :---: | :---: | :---: | :---: |
| $\mathrm{P}=57[\mathrm{w}] \mathrm{T}=\mathbf{2 4}\left[{ }^{\circ} \mathrm{C}\right]$ |  |  |  |
| PointsExperiment <br> Temperature | Simulation <br> Temperature | S.D. |  |
| 4 | 28.3 | 28.9 | 0.42 |
| 5 | 28.1 | 28.9 | 0.57 |
| 7 | 25.6 | 25.4 | 0.14 |
| 8 | 27.2 | 28.9 | 1.20 |
|  |  |  |  |
|  |  | RMS | 0.70 |


| Flow Simulation |  |  |  |
| :---: | :---: | :---: | :---: |
| $\mathrm{P}=57[\mathrm{w}] \mathrm{T}=24\left[{ }^{\circ} \mathrm{C}\right]$ |  |  |  |
| Points | Experiment <br> Temperature | Simulation <br> Temperature | S.D. |
| 4 | 28.3 | 28.47 | 0.12 |
| 5 | 28.1 | 28.44 | 0.24 |
| 7 | 25.6 | 24.90 | 0.49 |
| 8 | 27.4 | 27.83 | 0.30 |




Figure 18: Experiment No.0, 57[w] Graphs
After performing simulations on the Thermal and Flow Simulation tools, I decided to use the Flow Simulation tool for future simulations as it allows getting more accurate results. Discrepancies between Experiments and Simulation results are caused by:

- Difficulty to implement mesh on thin fins
- Fans connected close to the fins may cause a partly developed flow in a fins box
- Deviation resulted from Simulation tools

The Figure 19 shows results for the $57[w]$ temperature distribution:


Figure 19: Simulation Temperature Distribution
The Table 10 displays the Experiment No. 0 flow results:
Table 10: Experiment No. 0 flow Results

| Parameter | Maximum <br> value | Minimum <br> Value | Difference | Calculation <br> Value | See Figure |
| :--- | :--- | :--- | :---: | :--- | :---: |
| Flow Velocity <br> $[\mathrm{m} / \mathrm{sec}]$ | 3.218 | - | $\mathbf{3 . 2 1 8}$ | $\mathbf{4 . 0}$ | 20 |
| Pressure $\{\mathrm{Pa}]$ | 101324.65 | 101254.34 | $\mathbf{7 0 . 3}$ | $\mathbf{5 0}$ | 21 |
| Temperature $\left[{ }^{\circ} \mathrm{C}\right]$ | 27.6 | 23 | $\mathbf{4 . 6}$ | $\mathbf{4}$ | 22 |



Figure 20: Fluid Velocity


Figure 21: Pressure


Figure 22: Fluid Temperature

### 3.2. Vapor Chamber Thermal Properties Setting

This section describes the thermal properties of Vapor Chamber defined during the laboratory experiment (step 1), simulations (step 2), and statistical analysis (step 3).

- Step 1: Figure 23 below shows the TEC connected to the various parts of the Vapor Chamber on its hot side. Different power values were supplied to the TEC during the experiments. Thermocouples were mounted on the various points on the Vapor Chamber to find out its temperature distribution (see Figures 23 and 25).


Figure 23: Laboratory Experiment

- Step 2: Using the SolidWorks Flow Simulation tool, we defined temperature at the same points by simulating thermal properties of the Vapor Chamber. During the simulations, the Vapor Chamber was defined as orthotropic material. An orthotropic material has three mutually orthogonal twofold axes of rotational symmetry, so that its material properties are different along each axis.

Figure 24 shows coordinate system that we used in simulations. These simulations have been performed for various values of the thermal conductivity parameters ( $\mathrm{Kx}, \mathrm{Ky}, \mathrm{Kz}$ )


Figure 24: Coordinate System of Assembly

Step 3: To understand effective thermal properties of Vapor Chamber, I defined thermal conductivity parameters ( $K x, K y, K y$ ) by following the steps:

- Finding RMS (Root Mean Square) of standard deviation between experiment and simulation temperature by using the Excel application.
- Finding parameters of the following function $Y=f(\mathrm{Kz}, \mathrm{Kx}, \mathrm{Ky})$ by using the LabFit application, where $Y$ is the mentioned RMS of temperature differentials.
- Finding critical points of the $Y=f(K z, K x, K y)$ function with the goal to find value of $\mathrm{K}_{\mathrm{X}}, \mathrm{K}_{\mathrm{Y}}$ and $\mathrm{K}_{\mathrm{Z} \text { providing }}$ minimum RMS deviation between the measured and calculated data.


### 3.3. Experiment No. 1

The boundary conditions for Experiment No. 1 are shown in the Table 11 and Figure 25:
Table 11: Boundary Conditions for Experiment No. 1

| Thermodynamic parameters | Static Pressure: $101325.00[\mathrm{~Pa}]$ <br> Ambient Temperature: $23\left[{ }^{\circ} \mathrm{C}\right]$ |
| :--- | :--- |
| Material parameters | Default material: Aluminum $6061, \mathrm{~K}=167\left[\frac{\mathrm{~W}}{\mathrm{~m} \times{ }^{\circ} \mathrm{C}}\right]$ <br> Vapor Chamber material: TBD |
| Heat Power | $57[\mathrm{w}] \& 114[\mathrm{w}]$ |
| Contact Resistances | Thermal Conductivity $6\left[\frac{\mathrm{~W}}{\mathrm{~m} \times{ }^{\circ}}\right]$ <br> Tflex 220-0.02in( 0.51 mm$)-100 \mathrm{psi}$ <br> Faces: Vapor Chamber and Fins Box |



Figure 25: Thermocouples Points of Experiment No. 1

The experiment results are shown in the Table 12:
Table 12: Experiment No. 1 Results

| $\mathbf{I}[\mathbf{a m p}]$ | 5.7 | 7.6 |
| :--- | :---: | :---: |
| $\mathbf{V}[\mathbf{v}]$ | 10 | 15 |
| $\mathbf{P}[\mathbf{w}]$ | 57 | 114 |
| Points | $\mathbf{T}\left[{ }^{\circ} \mathbf{C}\right]$ | $\mathbf{T}\left[{ }^{\circ} \mathbf{C}\right]$ |
| P4 | 26.3 | 30.8 |
| P5 | 27.0 | 32.2 |
| P6 | 27.2 | 32.5 |
| P7 | 27.8 | 35.1 |
| P8 | 28.4 | 34.9 |
| P9 | 28.0 | 34.5 |
| P10 | 27.8 | 34.4 |
| P12 | 26.7 | 31.8 |
| P13 | 24.3 | 25.1 |
| P14 | 23.4 | 27.4 |

### 3.3.1. Experiment No. 1 with $\mathbf{P}=114[\mathrm{w}]$

The Table 13 shows the example of temperature analysis using the Excel application, where:

- Exper. Temp is an experiment temperature
- Simul. Temp is a Solid Works Flow simulation temperature
- S.D is a standard deviation between experiment and simulation temperatures
- RMS is a root mean square value of standard deviation

Table 13: Example of Temperature Analysis

|  | $K x=500 \mathrm{Kz=500} \mathrm{Ky=200}$ |  |  |  | $K x=500 \mathrm{Kz=500} \mathrm{Ky=1000}$ |  |  |  |  | $K x=500 \mathrm{Kz=500} \mathrm{Ky=1100}$ |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Exper <br> Temp <br> [ ${ }^{\circ} \mathrm{C}$ ] | Simul Temp [ ${ }^{\circ} \mathrm{C}$ ] | S.D | Aver |  | Exper Temp [ ${ }^{\circ} \mathrm{C}$ ] | Simul Temp [ ${ }^{\circ} \mathrm{C}$ ] | S.D | Aver |  | Exper Temp [ ${ }^{\circ} \mathrm{C}$ ] | Simul Temp [ ${ }^{\circ} \mathrm{C}$ ] | S.D | Aver |
| P4 | 30.8 | 32.9 | 1.5 | 31.9 | P4 | 30.8 | 33.0 | 1.5 | 31.9 | P4 | 30.8 | 33.0 | 1.5 | 31.9 |
| P5 | 32.2 | 32.9 | 0.5 | 32.5 | P5 | 32.2 | 32.9 | 0.5 | 32.6 | P5 | 32.2 | 32.9 | 0.5 | 32.6 |
| P6 | 32.5 | 28.6 | 2.7 | 30.6 | P6 | 32.5 | 28.7 | 2.7 | 30.6 | P6 | 32.5 | 28.7 | 2.7 | 30.6 |
| P7 | 35.1 | 34.5 | 0.5 | 34.8 | P7 | 35.1 | 34.3 | 0.6 | 34.7 | P7 | 35.1 | 34.2 | 0.6 | 34.7 |
| P8 | 34.9 | 34.7 | 0.2 | 34.8 | P8 | 34.9 | 34.7 | 0.2 | 34.8 | P8 | 34.9 | 34.7 | 0.2 | 34.8 |
| P9 | 34.5 | 35.4 | 0.7 | 35.0 | P9 | 34.5 | 35.2 | 0.5 | 34.9 | P9 | 34.5 | 35.2 | 0.5 | 34.9 |
| P10 | 34.4 | 35.6 | 0.8 | 35.0 | P10 | 34.4 | 35.4 | 0.7 | 34.9 | P10 | 34.4 | 35.4 | 0.7 | 34.9 |
| P12 | 31.8 | 32.1 | 0.2 | 31.9 | P12 | 31.8 | 32.2 | 0.3 | 32.0 | P12 | 31.8 | 32.2 | 0.3 | 32.0 |
| P13 | 25.1 | 25.8 | 0.5 | 25.4 | P13 | 25.1 | 25.8 | 0.5 | 25.4 | P13 | 25.1 | 25.8 | 0.5 | 25.4 |
| P14 | 27.4 | 31.2 | 2.7 | 29.3 | P14 | 27.4 | 31.4 | 2.8 | 29.4 | P14 | 27.4 | 31.4 | 2.8 | 29.4 |
|  |  | RMS | 1.3706 |  |  |  | RMS | 1.3889 |  |  |  | RMS | 1.3887 |  |

The Table 14 shows a summary of temperature analysis results, where:

- Kz - a length thermal conductivity $\left[\frac{\mathrm{W}}{\mathrm{m} \times{ }^{\circ} \mathrm{C}}\right]$
- $\mathrm{Kx}-\mathrm{a}$ width thermal conductivity $\left[\frac{\mathrm{W}}{\mathrm{m} \times{ }^{\circ} \mathrm{C}}\right]$
- Ky - a thickness thermal conductivity $\left[\frac{\mathrm{W}}{\mathrm{m} \times{ }^{\circ} \mathrm{C}}\right]$

Table 14: Summary of Experiment No. 1 with $P=114[w]$ Results

| No | $\boldsymbol{Y}_{\boldsymbol{R M S}}$ | $\mathbf{K z}$ | $\mathbf{K x}$ | $\mathbf{K y}$ |
| :---: | :---: | :---: | :---: | :---: |
| $\mathbf{1}$ | $\mathbf{1 . 3 7 0 6}$ | 500 | 500 | 200 |
| $\mathbf{2}$ | $\mathbf{1 . 3 9 4 6}$ | 500 | 500 | 300 |
| $\mathbf{3}$ | $\mathbf{1 . 3 9 2 3}$ | 500 | 500 | 400 |
| $\mathbf{4}$ | $\mathbf{1 . 3 9 0 3}$ | 500 | 500 | 600 |
| $\mathbf{5}$ | $\mathbf{1 . 3 8 8 9}$ | 500 | 500 | 1000 |
| $\mathbf{6}$ | $\mathbf{1 . 3 8 8 7}$ | 500 | 500 | 1100 |
| $\mathbf{7}$ | $\mathbf{1 . 3 8 8 6}$ | 500 | 500 | 1200 |
| $\mathbf{8}$ | $\mathbf{1 . 3 8 8 3}$ | 500 | 500 | 1500 |
| $\mathbf{9}$ | $\mathbf{1 . 3 8 8 0}$ | 500 | 500 | 2000 |
| $\mathbf{1 0}$ | $\mathbf{1 . 3 8 7 8}$ | 500 | 500 | 3000 |
| $\mathbf{1 1}$ | $\mathbf{1 . 3 8 7 6}$ | 500 | 500 | 3500 |
| $\mathbf{1 2}$ | $\mathbf{1 . 3 8 7 6}$ | 500 | 500 | 4000 |
| $\mathbf{1 3}$ | $\mathbf{1 . 3 8 7 5}$ | 500 | 500 | 5000 |


| No | $Y_{\text {RMS }}$ | Kz | Kx | Ky |
| :---: | :---: | :---: | :---: | :---: |
| 14 | 1.3872 | 500 | 500 | 10000 |
| 15 | 1.2435 | 1000 | 500 | 1100 |
| 16 | 1.2309 | 1200 | 500 | 1100 |
| 17 | 1.2265 | 1500 | 500 | 1100 |
| 18 | 1.2270 | 1600 | 500 | 1100 |
| 19 | 1.2282 | 1700 | 500 | 1100 |
| 20 | 1.2338 | 2000 | 500 | 1100 |
| 21 | 1.2589 | 3000 | 500 | 1100 |
| 22 | 1.2961 | 5000 | 500 | 1100 |
| 23 | 1.3392 | 10000 | 500 | 1100 |
| 24 | 1.2307 | 1500 | 200 | 1100 |
| 25 | 1.2159 | 1500 | 300 | 1100 |
| 26 | 1.2186 | 1500 | 400 | 1100 |
| 27 | 1.2265 | 1500 | 500 | 1100 |
| 28 | 1.2746 | 1500 | 1000 | 1100 |
| 29 | 1.3071 | 1500 | 1500 | 1100 |
| 30 | 1.3558 | 1500 | 3000 | 1100 |
| 31 | 1.3821 | 1500 | 5000 | 1100 |
| 32 | 1.4053 | 1500 | 10000 | 1100 |

I used the LAB Fit application for finding a function of $Y=f(\mathrm{Kz}, \mathrm{Kx}, \mathrm{Ky})$,
where Y has an RMS value.
Function No. 1 of Experiment No. 1 with $\mathrm{P}=114[\mathrm{w}]$ :

$$
\mathrm{Y}=\mathrm{A} \times \mathrm{Kz}^{2}+\mathrm{B} \times \mathrm{Kx}^{2}+\mathrm{C} \times \mathrm{Ky}^{2}+\mathrm{D} \times \mathrm{Kz}+\mathrm{E} \times \mathrm{Kx}+\mathrm{F} \times \mathrm{Ky}+\mathrm{G}
$$

Figure 26 shows an example, where mean parameters values are smaller than standard deviation values. It means that the function is wrong.


Figure 26: Function No. 1 Experiment No. 1 with $P=114[w]$
Function No. 2 of Experiment No. 1 with $\mathrm{P}=114[\mathrm{w}]$ :
$\mathrm{Y}=\mathrm{A} \times \mathrm{Kz}^{4}+\mathrm{B} \times \mathrm{Kx}^{4}+\mathrm{C} \times \mathrm{Ky}^{3}+\mathrm{D} \times \mathrm{Kz}^{2}+\mathrm{E} \times \mathrm{Kx}^{2}+\mathrm{F} \times \mathrm{Ky}^{2}+\mathrm{G} \times \mathrm{Kz}^{5}+\mathrm{H} \times \mathrm{Kx}^{5}+$ $+\mathrm{I} \times \mathrm{Ky}^{5}+\mathrm{J}$
Figure 27 shows that mean parameters values are higher than standard deviation values . It means that the function is right.


Figure 27: Function No. 2 Experiment No. 1 with $\mathbf{P}=114[w]$

The complete Function No. 2 is as follows:
$\mathrm{Y}=-\left(0.178 \times 10^{-15}\right) \times \mathrm{Kz}^{4}-\left(0.259 \times 10^{-15}\right) \times \mathrm{Kx}^{4}-\left(0.458 \times 10^{-11}\right) \times \mathrm{Ky}^{3}$
$-\left(0.257 \times 10^{-8}\right) \times \mathrm{Kz}^{2}+\left(0.722 \times 10^{-8}\right) \times \mathrm{Kx}^{2}+\left(0.232 \times 10^{-7}\right) \times \mathrm{Ky}^{2}+$
$+\left(0.207 \times 10^{-19}\right) \times \mathrm{Kz}^{5}+\left(0.198 \times 10^{-19}\right) \times \mathrm{Kx}^{5}+\left(0.236 \times 10^{-19}\right) \times \mathrm{Ky}^{5}$
+1.288
The Table 15 compares the values received from the Function No. 2 and RMS values.
Table 15: Function Values vs. RMS Values

| $\mathrm{K}_{\mathrm{z}}$ | $\mathrm{K}_{\mathrm{x}}$ | $\mathrm{K}_{\mathrm{y}}$ | Function Value | RMS Value |
| :---: | :---: | :---: | :---: | :---: |
| 1000 | 500 | 1100 | 1.309 | 1.27464 |

Calculation of critical points for $z$ axial:
$\mathrm{Y}^{\prime} \mathrm{z}=-4 \times\left(0.178 \times 10^{-15}\right) \times \mathrm{Kz}^{3}-2 \times\left(0.257 \times 10^{-8}\right) \times \mathrm{Kz}+5 \times\left(0.207 \times 10^{-19}\right) \times \mathrm{Kz}^{4}$

$$
=0
$$

$\frac{\mathrm{Kz}}{10^{-8}} \times\left(1.035 \times 10^{-11} \times \mathrm{Kz}^{3}-7.12 \times 10^{-8} \times \mathrm{Kz}^{2}-1.035\right)=0$
$K z \neq 0$
$1.035 \times 10^{-11} \times \mathrm{Kz}^{3}-7.12 \times 10^{-8} \times \mathrm{Kz}^{2}-1.035=0$
I used online cubic of Wolfram equation to solve the cubic equation (see Figure 28) for $z$ axial, where the results are as follows:

- $K_{\mathrm{z} 1}=8322.86$
- $\mathrm{K}_{\mathrm{z} 2}=-721.814-3390.29 \mathrm{i}$
- $\mathrm{K}_{\mathrm{z} 3}=-721.814+3390.29 \mathrm{i}$


Figure 28: Online Cubic Equation Solver
$\mathrm{Y}^{\prime \prime} \mathrm{z}=-12 \times\left(0.178 \times 10^{-15}\right) \times \mathrm{Kz}^{2}-2 \times\left(0.257 \times 10^{-8}\right)+20 \times\left(0.207 \times 10^{-19}\right) \times \mathrm{Kz}^{3}$ $\mathrm{Y}^{\prime \prime} \mathrm{z}(8322.86)=8.56 \times 10^{-8}>0-\min$

Calculation of critical points for $x$ axial:

$$
Y^{\prime} x=-4 \times\left(0.259 \times 10^{-15}\right) \times K x^{3}+2 \times\left(0.722 \times 10^{-8}\right) \times K x+5 \times\left(0.198 \times 10^{-19}\right)
$$

$$
\times K y^{4}=0
$$

$\frac{\mathrm{Kx}}{10^{-8}} \times\left(9.9 \times 10^{-12} \times \mathrm{Kz}^{3}-1.036 \times 10^{-7} \times \mathrm{Kz}+1.444\right)=0$
$\mathrm{Kx} \neq 0 \Rightarrow 9.9 \times 10^{-12} \times \mathrm{Kz}^{3}-1.036 \times 10^{-7} \times \mathrm{Kz}+1.444=0$
I used online cubic of Wolfram equation to solve the cubic equation (see Figure 29) for $z$ axial, where the results are as follows:

- $\mathrm{K}_{\mathrm{x} 1}=-3259.99$
- $\mathrm{K}_{\mathrm{x} 2}=5329.58$
- $K_{x 3}=8395.05$


Figure 29: Online Cubic Equation Solver

$$
\begin{aligned}
& \mathrm{Y}^{\prime \prime} \mathrm{x}=20 \times\left(0.110 \times 10^{-14}\right) \times \mathrm{Kx}^{3}-6 \times\left(0.182 \times 10^{-6}\right) \times \mathrm{Kx}+2 \times\left(0.716 \times 10^{-3}\right) \\
& \mathrm{Y}^{\prime \prime}(2857.82)= 20 \times\left(0.110 \times 10^{-14}\right) \times(2857.82)^{3}-6 \times\left(0.182 \times 10^{-6}\right) \times(2857.82)+2 \\
& \times\left(0.716 \times 10^{-3}\right)=-0.00111<0-\max
\end{aligned}
$$

$\mathrm{Y}^{\prime \prime}(8222.38)=0.004683<0-$ min.
Calculation of critical points for $y$ axial:
$\mathrm{Y}^{\prime \prime} \mathrm{x}=-12 \times\left(0.259 \times 10^{-15}\right) \times \mathrm{Kx}^{2}+2 \times\left(0.722 \times 10^{-8}\right)+20 \times\left(0.198 \times 10^{-19}\right) \times \mathrm{Ky}^{3}$
$\mathrm{Y}^{\prime \prime} \mathrm{x}(-3259.99)=-3.238 \times 10^{-8}<0-\max$
$\mathrm{Y}^{\prime \prime} \mathrm{x}(5329.58)=-1.39 \times 10^{-8}<0-\max$
$\mathrm{Y}^{\prime \prime} \mathrm{x}(8395.05)=2.97 \times 10^{-8}>0-\min$

$$
Y^{\prime} y=-3 \times\left(0.458 \times 10^{-11}\right) \times \mathrm{Ky}^{2}+2 \times\left(0.232 \times 10^{-7}\right) \times \mathrm{Ky}+5 \times\left(0.236 \times 10^{-19}\right)
$$

$$
\times \mathrm{Ky}^{4}=0
$$

$\frac{\mathrm{Ky}}{10^{-7}} \times\left(1.18 \times 10^{-12} \times \mathrm{Ky}^{3}-1.374 \times 10^{-4} \times \mathrm{Ky}+0.464\right)=0$

- $\mathrm{K}_{\mathrm{y} 1}=-12193.8$
- $\mathrm{K}_{\mathrm{y} 2}=3877.78$
- $\mathrm{K}_{\mathrm{y} 3}=8316$
$Y^{\prime \prime} y=-6 \times\left(0.458 \times 10^{-11}\right) \times \mathrm{Ky}+2 \times\left(0.232 \times 10^{-7}\right)+20 \times\left(0.236 \times 10^{-19}\right) \times \mathrm{Ky}^{3}$
$\mathrm{Y}^{\prime \prime} \mathrm{y}(-12193.8)=-8.09 \times 10^{-7}<0-\max$
$Y^{\prime \prime} y(3877.78)=-7.39 \times 10^{-8}<0-\max$
$Y^{\prime \prime} y(8316)=3.18 \times 10^{-7}>0-\min$
Table 16: Minimum of Function No.2, Experiment No. 1 with $P=114[w]$

| $\mathbf{K}_{\mathbf{z}}$ | $\mathbf{K}_{\mathbf{x}}$ | $\mathbf{K}_{\mathbf{y}}$ |
| :---: | :---: | :---: |
| 8322.86 | 8395.05 | 8316 |

### 3.3.2. Experiment No. 1 Results with $\mathbf{P}=57$ [w]

The summary of Experiment No. 1 with $\mathrm{P}=57$ [w] shown in the Table 17:
Table 17: Summary of Experiment No. 1 with $\mathrm{P}=57[\mathrm{w}]$ Results

| No | $Y_{\text {RMS }}$ | Kz | Kx | Ky |
| :---: | :---: | :---: | :---: | :---: |
| 1 | 0.8467 | 500 | 500 | 200 |
| 2 | 0.8454 | 500 | 500 | 1000 |
| 3 | 0.8453 | 500 | 500 | 1200 |
| 4 | 0.8452 | 500 | 500 | 1500 |
| 5 | 0.8451 | 500 | 500 | 1800 |
| 6 | 0.8451 | 500 | 500 | 2000 |
| 7 | 0.8450 | 500 | 500 | 2500 |
| 8 | 0.8450 | 500 | 500 | 3000 |
| 9 | 0.8449 | 500 | 500 | 4000 |
| 10 | 0.8448 | 500 | 500 | 5000 |
| 11 | 0.8448 | 500 | 500 | 10000 |
| 12 | 0.7970 | 200 | 500 | 1200 |
| 13 | 0.8453 | 1000 | 500 | 1200 |
| 14 | 0.8036 | 1200 | 500 | 1200 |
| 15 | 0.8034 | 1500 | 500 | 1200 |
| 16 | 0.8030 | 1600 | 500 | 1200 |
| 17 | 0.8032 | 1700 | 500 | 1200 |
| 18 | 0.8059 | 1800 | 500 | 1200 |
| 19 | 0.8061 | 2000 | 500 | 1200 |
| 20 | 0.8125 | 3000 | 500 | 1200 |
| 21 | 0.8213 | 5000 | 500 | 1200 |
| 22 | 0.8211 | 10000 | 500 | 1200 |
| 23 | 0.8015 | 1600 | 200 | 1200 |
| 24 | 0.7976 | 1600 | 300 | 1200 |
| 25 | 0.7989 | 1600 | 400 | 1200 |


| $\mathbf{N o}$ | $\boldsymbol{Y}_{\boldsymbol{R M S}}$ | $\mathbf{K z}$ | $\mathbf{K x}$ | Ky |
| :---: | :---: | :---: | :---: | :---: |
| $\mathbf{2 6}$ | $\mathbf{0 . 8 1 4 5}$ | 1600 | 500 | 1200 |
| $\mathbf{2 7}$ | $\mathbf{0 . 8 1 5 0}$ | 1600 | 800 | 1200 |
| $\mathbf{2 8}$ | $\mathbf{0 . 8 1 5 1}$ | 1600 | 1000 | 1200 |
| $\mathbf{2 9}$ | $\mathbf{0 . 8 1 9 5}$ | 1600 | 1500 | 1200 |
| $\mathbf{3 0}$ | $\mathbf{0 . 8 2 9 5}$ | 1600 | 2000 | 1200 |
| $\mathbf{3 1}$ | $\mathbf{0 . 8 3 6 6}$ | 1600 | 3000 | 1200 |
| $\mathbf{3 2}$ | $\mathbf{0 . 8 4 3 8}$ | 1600 | 5000 | 1200 |
| $\mathbf{3 3}$ | $\mathbf{0 . 8 5 0 5}$ | 1600 | 10000 | 1200 |

Function No. 3 of Experiment No. 1 with $\mathrm{P}=57[\mathrm{w}]$ :

$$
\begin{aligned}
\mathrm{Y}= & \mathrm{A} \times \mathrm{Kz}^{4}+\mathrm{B} \times \mathrm{Kx}^{5}+\mathrm{C} \times \mathrm{Ky}^{5}+\mathrm{D} \times \mathrm{Kz}^{3}+\mathrm{E} \times \mathrm{Kx}^{3}+\mathrm{F} \times \mathrm{Ky}^{3}+\mathrm{G} \times \mathrm{Kz}^{2}+ \\
& +\mathrm{H} \times \mathrm{Kx}^{2}+\mathrm{J} \times \mathrm{Ky}^{2}+\mathrm{J}
\end{aligned}
$$

Figure 30 shows that mean parameters values are higher than standard deviation values. It means that the function is right.


Figure 30: Function No. 3 Experiment No. 1 with $\mathbf{P}=57$ [w]
The complete Function No. 3 is as follows:
$Y=-\left(0.324 \times 10^{-15}\right) \times \mathrm{Kz}^{4}+\left(0.127 \times 10^{-19}\right) \times \mathrm{Kx}^{5}+\left(0.143 \times 10^{-19}\right) \times \mathrm{Ky}^{5}$
$+\left(0.491 \times 10^{-11}\right) \times \mathrm{Kz}^{3}-\left(0.237 \times 10^{-11}\right) \times \mathrm{Kx}^{3}-\left(0.254 \times 10^{-11}\right) \times \mathrm{Ky}^{3}-$
$-\left(0.168 \times 10^{-7}\right) \times \mathrm{Kz}^{2}+\left(0.114 \times 10^{-7}\right) \times \mathrm{Kx}^{2}+\left(0.113 \times 10^{-7}\right) \times \mathrm{Ky}^{2}$
$+0.820$
The Table 18 compares the values received from the Function No. 3 and RMS values.
Table 18: Function Values vs. RMS Values

| $\mathrm{K}_{\mathrm{z}}$ | $\mathrm{K}_{\mathrm{X}}$ | $\mathrm{K}_{\mathrm{y}}$ | Function Value | RMS Value |
| :---: | :---: | :---: | :---: | :---: |
| 1000 | 500 | 1200 | 0.816 | 0.845 |

Calculation of critical points for $z$ axial:
$Y^{\prime} z=-4 \times\left(0.324 \times 10^{-15}\right) \times \mathrm{Kz}^{3}+3 \times\left(0.491 \times 10^{-11}\right) \times \mathrm{Kz}^{2}-2 \times\left(0.168 \times 10^{-7}\right) \mathrm{Kz}$

$$
=0
$$

$\left.\frac{\mathrm{Kz}}{10^{-7}} \times\left(-1.296 \times 10^{-8}\right) \times \mathrm{Kz}^{2}+\left(1.473 \times 10^{-4}\right) \times \mathrm{Kz}-0.336\right)=0$
The calculation results are as follows:

- $\mathrm{K}_{\mathrm{z} 1}=3159.16$
- $K_{\mathrm{z} 2}=8206.58$
$\mathrm{Y}^{\prime \prime} \mathrm{z}=-12 \times\left(0.324 \times 10^{-15}\right) \times \mathrm{Kz}^{2}+6 \times\left(0.491 \times 10^{-11}\right) \times \mathrm{Kz}-2 \times\left(0.168 \times 10^{-7}\right)$
$\mathrm{Y}^{\prime \prime} \mathrm{z}(3159.16)=-5.4 \times 10^{-4}<0-\max$.
$\mathrm{Y}^{\prime} \mathrm{z}(8206.58)=2.07 \times 10^{-7}>0-\min$.
Calculation of critical points for $x$ axial:
$\mathrm{Y}^{\prime} \mathrm{x}=5 \times\left(0.127 \times 10^{-19}\right) \times \mathrm{Kx}^{4}-3 \times\left(0.237 \times 10^{-11}\right) \times \mathrm{Kx}^{2}+2 \times\left(0.114 \times 10^{-7}\right) \mathrm{Kx}=0$
$\frac{\mathrm{Kx}}{10^{-7}} \times\left(6.35 \times 10^{-13} \times \mathrm{Kx}^{3}-7.11 \times 10^{-5} \times \mathrm{Kx}+0.23\right)=0$
The calculation results are as follows:
- $\mathrm{K}_{\mathrm{x} 1}=-11930.2$
- $\mathrm{K}_{\mathrm{x} 2}=3679.95$
- $K_{x 3}=8250.21$
$\mathrm{Y}^{\prime \prime} \mathrm{x}=20 \times\left(0.127 \times 10^{-19}\right) \times \mathrm{Kx}^{3}-6 \times\left(0.237 \times 10^{-11}\right) \times \mathrm{Kx}+2 \times\left(0.114 \times 10^{-7}\right)$
$\mathrm{Y}^{\prime \prime}{ }^{x}(3679.95)=-1.6 \times 10^{-7}<0-\max$.
$\mathrm{Y}^{\prime \prime}{ }^{\prime}(8250.21)=4.81 \times 10^{-7}>0-\min$.

Calculation of critical points for $y$ axial:

$$
\begin{gathered}
\mathrm{Y}^{\prime} \mathrm{y}=5 \times\left(0.143 \times 10^{-19}\right) \times \mathrm{Ky}^{4}-3 \times\left(0.254 \times 10^{-11}\right) \times \mathrm{Ky}^{2}+2 \times\left(0.113 \times 10^{-7}\right) \times \mathrm{Ky} \\
\quad=0 \\
\frac{\mathrm{Ky}}{10^{-7}} \times\left(7.15 \times 10^{-13} \times \mathrm{Ky}^{3}-7.62 \times 10^{-5} \times \mathrm{Ky}+0.226\right)=0
\end{gathered}
$$

The calculation results are as follows:

- $\mathrm{K}_{\mathrm{y} 1}=-11571.1$
- $\mathrm{K}_{\mathrm{y} 2}=3304.45$
- $K_{y 2}=8266.64$
$\mathrm{Y}^{\prime \prime} \mathrm{y}=20 \times\left(0.143 \times 10^{-19}\right) \times \mathrm{Ky}^{3}-6 \times\left(0.254 \times 10^{-11}\right) \times \mathrm{Ky}+2 \times\left(0.113 \times 10^{-7}\right)$
$\mathrm{Y}^{\prime \prime} \mathrm{y}(3304.45)=-1.8 \times 10^{-7}<0-\max$.
$\mathrm{Y}^{\prime}{ }^{\prime} \mathrm{y}(8266.64)=5.67 \times 10^{-7}>0-\min$.
Table 19: Minimum of Function No.3, Experiment 1 with $P=57[w]$

| $\mathbf{K}_{\mathbf{z}}$ | $\mathbf{K}_{\mathbf{x}}$ | $\mathbf{K}_{\mathbf{y}}$ |
| :---: | :---: | :---: |
| 8206.58 | 8250.21 | 8266.64 |

### 3.4. Experiment No.2.

The boundary conditions for Experiment No. 2 are shown in the Table 20 and Figure 31:
Table 20: Boundary Condition of Experiment No. 2

| Thermodynamic parameters | Static Pressure: $101325.00[\mathrm{~Pa}]$ <br> Ambient Temperature: $24\left[{ }^{\circ} \mathrm{C}\right]$ |
| :--- | :--- |
| Material parameters | Default material: Aluminum $6061, \mathrm{~K}=167\left[\frac{\mathrm{~W}}{\mathrm{~m} \times{ }^{\circ} \mathrm{C}}\right]$ <br> Vapor Chamber material: TBD |
| Heat power | $110.72[\mathrm{w}], 80.94[\mathrm{w}] \& 52[\mathrm{w}]$ |
| Contact Resistances | Thermal Conductivity $6\left[\frac{\mathrm{~W}}{\mathrm{~m} \times{ }^{\circ} \mathrm{C}}\right]$ <br> Tflex 220-0.02in $(0.51 \mathrm{~mm})-100 \mathrm{psi}$ <br> Faces: Vapor Chamber and Fins Box |



Figure 31: Thermo-Coupler Points of Experiment No. 2
The summary of Experiment No. 2 shown in the Table 21:

Table 21: Experiment No. 2 Results

| $\mathrm{I}[\mathrm{amp}]$ | 6.4 | 5.7 | 5.2 |
| :--- | :---: | :---: | :---: |
| $\mathrm{~V}[\mathbf{v}]$ | 17.3 | 14.2 | 10 |
| $\mathrm{P}[\mathbf{w}]$ | 110.72 | 80.94 | 52 |
| Points | $\mathrm{T}\left[{ }^{\circ} \mathrm{C}\right]$ | $\mathrm{T}\left[{ }^{\circ} \mathrm{C}\right]$ | $\mathrm{T}\left[{ }^{\circ} \mathrm{C}\right]$ |
| P 4 | 32.0 | 29.7 | 29.9 |
| P 5 | 32.5 | 30.1 | 30.3 |
| P 6 | 38.6 | 34.1 | 32.3 |
| P 8 | 35.5 | 32.1 | 30.7 |
| P 9 | 36.2 | 32.2 | 30.9 |
| P 10 | 32.5 | 30.2 | 29.4 |
| P 12 | 32.2 | 29.8 | 29.8 |
| P 13 | 27.2 | 26.0 | 27.9 |
| P 14 | 28.8 | 27.2 | 27.0 |

### 3.4.1. Results for Experiment No. 2 with $\mathbf{P}=110.72$ [w]

The summary of Experiment No. 2 with $\mathrm{P}=110.72$ [w] shown in the Table 22:
Table 22: Summary of Experiment No. 2 with $\mathbf{P}=110.72$ [w] Results

| No | $\boldsymbol{Y}_{\text {RMS }}$ | Kz | $\mathbf{K x}$ | $\mathbf{K y}$ |
| :---: | :---: | :---: | :---: | :---: |
| $\mathbf{1}$ | $\mathbf{1 . 8 0 1 8}$ | 500 | 500 | 200 |
| $\mathbf{2}$ | $\mathbf{1 . 8 7 9 6}$ | 500 | 500 | 1000 |
| $\mathbf{3}$ | $\mathbf{1 . 8 8 5 0}$ | 500 | 500 | 1200 |
| $\mathbf{4}$ | $\mathbf{1 . 8 9 1 0}$ | 500 | 500 | 1500 |
| $\mathbf{5}$ | $\mathbf{1 . 9 0 5 0}$ | 500 | 500 | 2000 |
| $\mathbf{6}$ | $\mathbf{1 . 9 0 5 0}$ | 500 | 500 | 3000 |
| $\mathbf{7}$ | $\mathbf{1 . 9 0 7 3}$ | 500 | 500 | 3500 |
| $\mathbf{8}$ | $\mathbf{1 . 9 0 9 1}$ | 500 | 500 | 4000 |
| $\mathbf{9}$ | $\mathbf{1 . 9 1 0 0}$ | 500 | 500 | 5000 |
| $\mathbf{1 0}$ | $\mathbf{1 . 9 1 5 6}$ | 500 | 500 | 10000 |
| $\mathbf{1 1}$ | $\mathbf{1 . 9 1 7 7}$ | 500 | 500 | 15000 |
| $\mathbf{1 2}$ | $\mathbf{1 . 9 6 2 0}$ | 1000 | 500 | 1200 |


| No | $Y_{\text {RMS }}$ | Kz | Kx | Ky |
| :---: | :---: | :---: | :---: | :---: |
| 13 | 2.0662 | 2000 | 500 | 1200 |
| 14 | 2.1004 | 2500 | 500 | 1200 |
| 15 | 2.1275 | 3000 | 500 | 1200 |
| 16 | 2.1495 | 3500 | 500 | 1200 |
| 17 | 2.1677 | 4000 | 500 | 1200 |
| 18 | 2.1963 | 5000 | 500 | 1200 |
| 19 | 2.2352 | 7000 | 500 | 1200 |
| 20 | 2.2606 | 9000 | 500 | 1200 |
| 21 | 2.0427 | 10000 | 500 | 1200 |
| 22 | 2.2788 | 11000 | 500 | 1200 |
| 23 | 2.2861 | 12000 | 500 | 1200 |
| 24 | 2.3032 | 15000 | 500 | 1200 |
| 25 | 2.0233 | 3000 | 200 | 1200 |
| 26 | 2.1037 | 3000 | 400 | 1200 |
| 27 | 2.1275 | 3000 | 500 | 1200 |
| 28 | 2.1454 | 3000 | 600 | 1200 |
| 29 | 2.1706 | 3000 | 800 | 1200 |
| 30 | 2.1874 | 3000 | 1000 | 1200 |
| 31 | 2.2121 | 3000 | 1500 | 1200 |
| 32 | 2.2254 | 3000 | 2000 | 1200 |
| 33 | 2.2390 | 3000 | 3000 | 1200 |
| 34 | 2.2460 | 3000 | 4000 | 1200 |
| 35 | 2.2500 | 3000 | 5000 | 1200 |
| 36 | 2.4575 | 3000 | 10000 | 1200 |

Function No. 4 of Experiment No. 2 with $\mathrm{P}=110.72$ [w]:
$\mathrm{Y}=\mathrm{A} \times \mathrm{Kz}^{4}+\mathrm{B} \times \mathrm{Kx}^{4}+\mathrm{C} \times \mathrm{Ky}^{4}+\mathrm{D} \times \mathrm{Kz}^{3}+\mathrm{E} \times \mathrm{Kx}^{3}+\mathrm{F} \times \mathrm{Ky}^{3}+\mathrm{G} \times \mathrm{Kz}+\mathrm{H} \times \mathrm{Kx}+\mathrm{I} \times$ $\times \mathrm{Ky}+\mathrm{J}$

Figure 32 shows that mean parameters values are higher than standard deviation values . It means that the function is right.


Figure 32: Function No. 4 Experiment No. 2 with $\mathbf{P}=110.72$ [w]
The complete Function No. 4 is as follows:

$$
\begin{aligned}
& Y=\left(0.617 \times 10^{-16}\right) \times \mathrm{Kz}^{4}+\left(0.411 \times 10^{-15}\right) \times \mathrm{Kx}^{4}+\left(0.199 \times 10^{-16}\right) \times \mathrm{Ky}^{4}- \\
& -\left(0.122 \times 10^{-11}\right) \times \mathrm{Kz}^{3}-\left(0.473 \times 10^{-11}\right) \times \mathrm{Kx}^{3}-\left(0.523 \times 10^{-12}\right) \times \mathrm{Ky}^{3}+ \\
& +\left(0.982 \times 10^{-4}\right) \times \mathrm{Kz}+\left(0.101 \times 10^{-3}\right) \times \mathrm{Kx}+\left(0.351 \times 10^{-8}\right) \times \mathrm{Ky}^{2}+ \\
& +1.79
\end{aligned}
$$

The Table 23 compares the values received from the Function No. 4 and RMS values.
Table 23: Function Values vs. RMS Values

| $\mathrm{K}_{\mathrm{z}}$ | $\mathrm{K}_{\mathrm{x}}$ | $\mathrm{K}_{\mathrm{y}}$ | Function Value | RMS Value |
| :---: | :---: | :---: | :---: | :---: |
| 1000 | 500 | 1200 | 1.941668 | 1.962 |

Calculation of critical points for $z$ axial:

$$
\begin{aligned}
& Y^{\prime} z=4 \times\left(0.617 \times 10^{-16}\right) \times \mathrm{Kz}^{3}-3 \times\left(0.122 \times 10^{-11}\right) \times \mathrm{Kz}^{2}+\left(0.982 \times 10^{-4}\right)=0 \\
& \frac{1}{10^{-4}} \times\left(\left(2.468 \times 10^{-12}\right) \times \mathrm{Kz}^{3}-\left(3.66 \times 10^{-8}\right) \times \mathrm{Kz}^{2}+0.982\right)=0
\end{aligned}
$$

The calculation results are as follows:

- $\mathrm{K}_{\mathrm{z} 1}=-4533.12$
- $\mathrm{K}_{\mathrm{z} 2}=7240.92$
- $\mathrm{K}_{\mathrm{z} 3}=12122$
$\mathrm{Y}^{\prime \prime} \mathrm{z}=12 \times\left(0.617 \times 10^{-16}\right) \times \mathrm{Kz}^{2}-6 \times\left(0.122 \times 10^{-11}\right) \times \mathrm{Kz}$
$\mathrm{Y}^{\prime \prime} \mathrm{z}(-4533.12)=4.84 \times 10^{-8}>0-$ max.
$\mathrm{Y}^{\prime \prime} \mathrm{z}(7240.92)=-1.42 \times 10^{-8}<0-\max$.
$Y^{\prime \prime} \mathrm{z}(12122)=2 \times 10^{-8}>-\min$.
Calculation of critical points for $x$ axial:
$Y^{\prime} \mathrm{x}=4 \times\left(0.411 \times 10^{-15}\right) \times \mathrm{Kx}^{3}-3 \times\left(0.473 \times 10^{-11}\right) \times \mathrm{Kx}^{2}+\left(0.101 \times 10^{-3}\right)=0$
$\frac{1}{10^{-3}} \times\left(\left(1.644 \times 10^{-12}\right) \times \mathrm{Kx}^{3}-\left(1.42 \times 10^{-8}\right) \times \mathrm{Kx}+0.101\right)=0$
The calculation results are as follows:
- $\mathrm{K}_{\mathrm{x} 1}=-2363.2$
- $\mathrm{K}_{\mathrm{x} 2}=3437.11$
- $K_{\mathrm{x} 3}=7563.56$
$\mathrm{Y}^{\prime \prime} \mathrm{x}=12 \times\left(0.199 \times 10^{-16}\right) \times \mathrm{Kx}^{2}-6 \times\left(0.473 \times 10^{-11}\right) \times \mathrm{Kx}$
$\mathrm{Y}^{\prime \prime} \mathrm{x}(-2363.2)=9.5 \times 10^{-8}>0-\max$.
$\mathrm{Y}^{\prime \prime} \mathrm{x}(3437.11)=-3.9 \times 10^{-8}<0-\max$.
$\mathrm{Y}^{\prime \prime} \mathrm{x}(7563.56)=6.7 \times 10^{-8}>-$ min.
Calculation of critical points for $y$ axial:
$Y^{\prime} y=4 \times\left(0.199 \times 10^{-16}\right) \times \mathrm{Ky}^{3}-3 \times\left(0.523 \times 10^{-12}\right) \times \mathrm{Ky}^{2}+2 \times\left(0.351 \times 10^{-8}\right) \times \mathrm{Ky}$ $=0$
$\frac{1}{\mathrm{Ky} \times 10^{-8}} \times\left(\left(7.96 \times 10^{-9}\right) \times \mathrm{Ky}^{2}-\left(1.57 \times 10^{-4}\right) \times \mathrm{Ky}^{2}+0.702\right)=0$
The calculation results are as follows:
- $\mathrm{K}_{\mathrm{y} 1}=6851.11$
- $\mathrm{K}_{\mathrm{y} 3}=12872.5$
$Y^{\prime \prime} y=12 \times\left(0.199 \times 10^{-16}\right) \times \mathrm{Ky}^{2}-6 \times\left(0.523 \times 10^{-12}\right) \times \mathrm{Ky}+2 \times\left(0.351 \times 10^{-8}\right)$
$Y^{\prime \prime} y(6851.11)=-3.27 \times 10^{-9}<0-\max$.
$\mathrm{Y}^{\prime \prime} \mathrm{y}(12872.5)=6.19 \times 10^{-9}>0-\min$.

Table 24: Summary of Function No.4, Experiment No. 2 with $P=110.72$ [w]

| $\mathbf{K z}$ | $\mathbf{K} \boldsymbol{x}$ | $\mathbf{K} \boldsymbol{y}$ |
| :---: | :---: | :---: |
| 12122 | 7563.56 | 12872.5 |

### 3.4.2. Results for Experiment No. 2 with $\mathbf{P}=\mathbf{8 0 . 9 4}$ [w]

The summary of Experiment No. 2 with $\mathrm{P}=80.94$ [w] shown in the Table 25:
Table 25: Summary of Experiment No. 2 with $\mathrm{P}=80.94[\mathrm{w}]$ Results

| $\mathbf{N o}$ | $\boldsymbol{Y}_{\boldsymbol{R M S}}$ | $\mathbf{K z}$ | $\mathbf{K x}$ | $\mathbf{K y}$ |
| :---: | :---: | :---: | :---: | :---: |
| $\mathbf{1}$ | $\mathbf{1 . 3 8 3 9}$ | 500 | 500 | 200 |
| $\mathbf{2}$ | $\mathbf{1 . 4 2 5 8}$ | 500 | 500 | 600 |
| $\mathbf{3}$ | $\mathbf{1 . 4 3 8 7}$ | 500 | 500 | 1000 |
| $\mathbf{4}$ | $\mathbf{1 . 4 4 2 6}$ | 500 | 500 | 1200 |
| $\mathbf{5}$ | $\mathbf{1 . 4 4 6 8}$ | 500 | 500 | 1500 |
| $\mathbf{6}$ | $\mathbf{1 . 4 5 1 4}$ | 500 | 500 | 2000 |
| $\mathbf{7}$ | $\mathbf{1 . 4 5 6 7}$ | 500 | 500 | 3000 |
| $\mathbf{8}$ | $\mathbf{1 . 4 5 8 2}$ | 500 | 500 | 3500 |
| $\mathbf{9}$ | $\mathbf{1 . 4 5 9 3}$ | 500 | 500 | 4000 |
| $\mathbf{1 0}$ | $\mathbf{1 . 4 6 1 2}$ | 500 | 500 | 5000 |
| $\mathbf{1 1}$ | $\mathbf{1 . 4 6 5 3}$ | 500 | 500 | 10000 |
| $\mathbf{1 2}$ | $\mathbf{1 . 4 5 5 6}$ | 200 | 500 | 1200 |
| $\mathbf{1 3}$ | $\mathbf{1 . 4 7 6 2}$ | 800 | 500 | 1200 |
| $\mathbf{1 4}$ | $\mathbf{1 . 5 7 3 3}$ | 2000 | 500 | 1200 |


| $\mathbf{N o}$ | $\boldsymbol{Y}_{\boldsymbol{R M S}}$ | $\mathbf{K z}$ | Kx | Ky |
| :---: | :---: | :---: | :---: | :---: |
| $\mathbf{1 5}$ | $\mathbf{1 . 6 1 6 8}$ | 3000 | 500 | 1200 |
| $\mathbf{1 6}$ | $\mathbf{1 . 6 6 3 7}$ | 5000 | 500 | 1200 |
| $\mathbf{1 7}$ | $\mathbf{1 . 6 9 0 9}$ | 7000 | 500 | 1200 |
| $\mathbf{1 8}$ | $\mathbf{1 . 7 1 5 3}$ | 10000 | 500 | 1200 |
| $\mathbf{1 9}$ | $\mathbf{1 . 7 2 6 0}$ | 12000 | 500 | 1200 |
| $\mathbf{2 0}$ | $\mathbf{1 . 7 3 0 5}$ | 13000 | 500 | 1200 |
| $\mathbf{2 1}$ | $\mathbf{1 . 7 3 4 5}$ | 14000 | 500 | 1200 |
| $\mathbf{2 2}$ | $\mathbf{1 . 6 3 8 6}$ | 15000 | 500 | 1200 |
| $\mathbf{2 3}$ | $\mathbf{1 . 5 5 8 3}$ | 17000 | 500 | 1200 |
| $\mathbf{2 4}$ | $\mathbf{1 . 7 4 6 5}$ | 18000 | 500 | 1200 |
| $\mathbf{2 5}$ | $\mathbf{1 . 7 5 1 0}$ | 20000 | 500 | 1200 |
| $\mathbf{2 6}$ | $\mathbf{1 . 7 0 2 3}$ | 14000 | 300 | 1200 |
| $\mathbf{2 7}$ | $\mathbf{1 . 7 4 4 9}$ | 14000 | 600 | 1200 |
| $\mathbf{2 8}$ | $\mathbf{1 . 7 6 0 2}$ | 14000 | 800 | 1200 |
| $\mathbf{2 9}$ | $\mathbf{1 . 7 7 0 7}$ | 14000 | 1000 | 1200 |
| $\mathbf{3 0}$ | $\mathbf{1 . 7 8 6 9}$ | 14000 | 1500 | 1200 |
| $\mathbf{3 1}$ | $\mathbf{1 . 7 8 7 0}$ | 14000 | 2000 | 1200 |
| $\mathbf{3 2}$ | $\mathbf{1 . 8 0 7 1}$ | 14000 | 3000 | 1200 |
| $\mathbf{3 3}$ | $\mathbf{1 . 8 1 2 9}$ | 14000 | 4000 | 1200 |
| $\mathbf{3 4}$ | $\mathbf{1 . 8 1 6 5}$ | 14000 | 5000 | 1200 |
| $\mathbf{3 5}$ | $\mathbf{1 . 8 2 4 0}$ | 14000 | 10000 | 1200 |
| $\mathbf{3 6}$ | $\mathbf{1 . 8 2 6 3}$ | 14000 | 15000 | 1200 |
|  |  |  |  |  |

Function No. 5 of Experiment No. 2 with $\mathrm{P}=80.94$ [w]:
$\mathrm{Y}=\mathrm{A} \times \mathrm{Kz}^{3}+\mathrm{B} \times \mathrm{Kx}^{3}+\mathrm{C} \times \mathrm{Ky}^{3}+\mathrm{D} \times \mathrm{Kz}^{2}+\mathrm{E} \times \mathrm{Kx}^{2}+\mathrm{F} \times \mathrm{Ky}^{2}+\mathrm{G} \times \mathrm{Kz}^{4}+\mathrm{H} \times \mathrm{Kx}+\mathrm{I}$ $\times \times \mathrm{Ky}+\mathrm{J}$

Figure 33 shows that mean parameters values are higher than standard deviation values . It means that the function is right.


Figure 33: Function No. 5 Experiment No. 2 with $P=80.94$ [w]
The complete Function No. 5 is as follows:
$\mathrm{Y}=-\left(0.118 \times 10^{-11}\right) \times \mathrm{Kz}^{3}+\left(0.925 \times 10^{-12}\right) \times \mathrm{Kx}^{3}+\left(0.309 \times 10^{-11}\right) \times \mathrm{Ky}^{3}+$ $+\left(0.106 \times 10^{-7}\right) \times \mathrm{Kz}^{2}-\left(0.245 \times 10^{-7}\right) \times \mathrm{Kx}^{2}-\left(0.443 \times 10^{-7}\right) \times \mathrm{Ky}^{2}+$ $+\left(0.337 \times 10^{-16}\right) \times \mathrm{Kz}^{4}+\left(0.184 \times 10^{-3}\right) \times \mathrm{Kx}+\left(0.138 \times 10^{-3}\right) \times \mathrm{Ky}+1.34$

The Table 26 compares the values received from the Function No. 5 and RMS values.
Table 26: Function Values vs. RMS Values

| $\mathrm{K}_{\mathrm{z}}$ | $\mathrm{K}_{\mathrm{x}}$ | $\mathrm{K}_{\mathrm{y}}$ | Function Value | RMS Value |
| :---: | :---: | :---: | :---: | :---: |
| 2000 | 500 | 1200 | 1.566637 | 1.5733 |

Calculation of critical points for $z$ axial:

$$
\begin{aligned}
\mathrm{Y}^{\prime} \mathrm{z}=-3 \times( & \left.0.118 \times 10^{-11}\right) \times \mathrm{Kz}^{2}+2 \times\left(0.106 \times 10^{-7}\right) \times \mathrm{Kz}+4 \times\left(0.337 \times 10^{-16}\right) \times \mathrm{Kz}^{3} \\
& =0
\end{aligned}
$$

$\frac{\mathrm{Kz}}{10^{-7}} \times\left(\left(1.348 \times 10^{-10}\right) \times \mathrm{Kz}^{2}-\left(3.54 \times 10^{-5}\right) \times \mathrm{Kz}+0.212\right)=0$
The calculation results are as follows:

- $\mathrm{K}_{\mathrm{z} 1}=6131.88$
- $K_{\mathrm{z} 2}=16647.9$
$\mathrm{Y}^{\prime \prime} \mathrm{z}=-6 \times\left(0.118 \times 10^{-11}\right) \times \mathrm{Kz}+2 \times\left(0.106 \times 10^{-7}\right)+12 \times\left(0.337 \times 10^{-16}\right) \times \mathrm{Kz}^{2}$
$\mathrm{Y}^{\prime \prime} \mathrm{z}(6131.88)=-7 \times 10^{-9}<0-\max$
$\mathrm{Y}^{\prime \prime} \mathrm{z}(16647.9)=1.54 \times 10^{-8}>0-\min$
Calculation of critical points for $x$ axial:
$Y^{\prime} \mathrm{x}=3 \times\left(0.925 \times 10^{-12}\right) \times \mathrm{Kx}^{2}-2 \times\left(0.245 \times 10^{-7}\right) \times \mathrm{Kx}+\left(0.184 \times 10^{-3}\right)=0$
$\frac{1}{10^{-3}} \times\left(\left(2.775 \times 10^{-9}\right) \times \mathrm{Kx}^{2}-\left(4.9 \times 10^{-5}\right) \times \mathrm{Kx}+0.184\right)=0$
The calculation results are as follows:
- $\mathrm{K}_{\mathrm{x} 1}=5416.8$
- $K_{\mathrm{x} 2}=12240.8$
$\mathrm{Y}^{\prime \prime} \mathrm{x}=6 \times\left(0.925 \times 10^{-12}\right) \times \mathrm{Kx}-2 \times\left(0.245 \times 10^{-7}\right)$
$\mathrm{Y}^{\prime \prime} \mathrm{x}(5416.8)=-1.9 \times 10^{-8}<0-\max$
$\mathrm{Y}^{\prime \prime} \mathrm{x}(12240.8)=1.89 \times 10^{-8}>0-\min$

Calculation of critical points for $y$ axial:
$Y^{\prime} y=3 \times\left(0.309 \times 10^{-11}\right) \times \mathrm{Ky}^{2}-2 \times\left(0.443 \times 10^{-7}\right) \times \mathrm{Ky}+\left(0.138 \times 10^{-3}\right)=0$
$\frac{1}{10^{-3}} \times\left(\left(9.27 \times 10^{-9}\right) \times \mathrm{Ky}^{2}-\left(8.86 \times 10^{-5}\right) \times \mathrm{Ky}+0.138\right)=0$
The calculation results are as follows:

- $\mathrm{K}_{\mathrm{y} 1}=1944.31$
- $K_{y 2}=7656.55$
$Y^{\prime \prime} y=6 \times\left(0.309 \times 10^{-11}\right) \times \mathrm{Ky}-2 \times\left(0.443 \times 10^{-7}\right)$
$Y^{\prime \prime} y(1944.31)=-5.3 \times 10^{-8}<0-\max$
$Y^{\prime \prime} y(7656.55)=5.2 \times 10^{-8}>0-\min$
Table 27: Minimum of Function No.5, Experiment No. 2 with $P=80.94[w]$

| $\mathbf{K} \boldsymbol{z}$ | $\mathbf{K} \boldsymbol{x}$ | $\mathbf{K} \boldsymbol{y}$ |
| :---: | :---: | :---: |
| 16647.9 | 12240.8 | 7656.55 |

### 3.4.3. Results for Experiment No. 2 with $\mathbf{P}=52$ [w]

The summary of Experiment No. 2 with $\mathrm{P}=52$ [w] shown in the Table 28:
Table 28: Summary of Experiment No. 2 with $\mathrm{P}=52[\mathrm{w}]$ Results

| No | $Y_{\text {RMS }}$ | Kz | Kx | Kу |
| :---: | :---: | :---: | :---: | :---: |
| 1 | 0.8146 | 500 | 500 | 200 |
| 2 | 0.7962 | 500 | 500 | 1000 |
| 3 | 0.7952 | 500 | 500 | 1200 |
| 4 | 0.7940 | 500 | 500 | 1500 |
| 5 | 0.7928 | 500 | 500 | 1800 |
| 6 | 0.7933 | 500 | 500 | 2000 |
| 7 | 0.7916 | 500 | 500 | 3000 |
| 8 | 0.7908 | 500 | 500 | 4000 |
| 9 | 0.7904 | 500 | 500 | 5000 |
| 10 | 0.7894 | 500 | 500 | 10000 |
| 11 | 0.7700 | 800 | 500 | 1200 |
| 12 | 0.7666 | 900 | 500 | 1200 |
| 13 | 0.7643 | 1000 | 500 | 1200 |
| 14 | 0.7635 | 1800 | 500 | 1200 |
| 15 | 0.7650 | 2000 | 500 | 1200 |
| 16 | 0.7750 | 3000 | 500 | 1200 |
| 17 | 0.7784 | 3400 | 500 | 1200 |
| 18 | 0.7792 | 3500 | 500 | 1200 |
| 19 | 0.7898 | 5000 | 500 | 1200 |
| 20 | 0.7670 | 10000 | 500 | 1200 |
| 21 | 0.7486 | 1000 | 300 | 1200 |
| 22 | 0.7589 | 1000 | 400 | 1200 |
| 23 | 0.7835 | 1000 | 1000 | 1200 |
| 24 | 0.7910 | 1000 | 1500 | 1200 |
| 25 | 0.7936 | 1000 | 1800 | 1200 |


| $\mathbf{N o}$ | $\boldsymbol{Y}_{\boldsymbol{R M S}}$ | $\mathbf{K z}$ | $\mathbf{K x}$ | Ky |
| :---: | :---: | :---: | :---: | :---: |
| $\mathbf{2 6}$ | $\mathbf{0 . 7 9 4 8}$ | 1000 | 2000 | 1200 |
| $\mathbf{2 7}$ | $\mathbf{0 . 7 9 8 4}$ | 1000 | 3000 | 1200 |
| $\mathbf{2 8}$ | $\mathbf{0 . 7 9 9 5}$ | 1000 | 4000 | 1200 |
| $\mathbf{2 9}$ | $\mathbf{0 . 8 0 0 0}$ | 1000 | 5000 | 1200 |
| $\mathbf{3 0}$ | $\mathbf{0 . 8 0 3 5}$ | 1000 | 10000 | 1200 |

Function No. 6 of Experiment No. 2 with $\mathrm{P}=52$ [w]:

$$
\begin{aligned}
& \mathrm{Y}=\mathrm{A} \times \mathrm{Kz}^{4}+ \mathrm{B} \times \mathrm{Kx}^{4}+\mathrm{C} \times \mathrm{Ky}^{4}+\mathrm{D} \times \mathrm{Kz}^{2}+\mathrm{E} \times \mathrm{Kx}^{2}+\mathrm{F} \times \mathrm{Ky}^{2}+\mathrm{G} \times \mathrm{Kz}+\mathrm{H} \times \mathrm{Kx}+\mathrm{I} \times \\
& \times \mathrm{Ky}+\mathrm{J}
\end{aligned}
$$

Figure 34 shows that mean parameters values are higher than standard deviation values . It means that the function is right.


Figure 34: Function No. 6 Experiment No. 2 with $\mathbf{P}=52$ [w]
The complete Function No. 6 is as follows:
$\mathrm{Y}=-\left(0.591 \times 10^{-16}\right) \times \mathrm{Kz}^{4}+\left(0.189 \times 10^{-16}\right) \times \mathrm{Kx}^{4}-\left(0.204 \times 10^{-16}\right) \times \mathrm{Ky}^{4}$ $+\left(0.971 \times 10^{-8}\right) \times \mathrm{Kz}^{2}-\left(0.383 \times 10^{-8}\right) \times \mathrm{Kx}^{2}+\left(0.360 \times 10^{-8}\right) \times \mathrm{Ky}^{2}-$ $-\left(0.422 \times 10^{-4}\right) \times \mathrm{Kz}+\left(0.233 \times 10^{-4}\right) \times \mathrm{Kx}-\left(0.171 \times 10^{-4}\right) \times \mathrm{Ky}$ $+0.812$

The Table 29 compares the values received from the Function No. 5 and RMS values.
Table 29: Function Values vs. RMS Values

| $\mathrm{K}_{\mathrm{z}}$ | $\mathrm{K}_{\mathrm{x}}$ | $\mathrm{K}_{\mathrm{y}}$ | Function Value | RMS Value |
| :---: | :---: | :---: | :---: | :---: |
| 1000 | 1000 | 1200 | 0.764 | 0.7589 |

Calculation of critical points for $z$ axial:
$Y^{\prime} z=-4 \times\left(0.591 \times 10^{-16}\right) \times \mathrm{Kz}^{3}+2 \times\left(0.971 \times 10^{-8}\right) \times \mathrm{Kz}-\left(0.422 \times 10^{-4}\right)=0$
$\frac{-1}{10^{-4}} \times\left(2.364 \times 10^{-12} \times \mathrm{Kz}^{3}-1.942 \times 10^{-4} \times \mathrm{Kz}+0.422\right)=0$
The calculation results are as follows:

- $\mathrm{K}_{\mathrm{z} 1}=-10000$
- $\mathrm{K}_{\mathrm{z} 2}=2326.26$
- $\mathrm{K}_{\mathrm{z} 3}=7673.74$
$\mathrm{Y}^{\prime \prime} \mathrm{z}=-12 \times\left(0.591 \times 10^{-16}\right) \times \mathrm{Kz}^{2}+2 \times\left(0.971 \times 10^{-8}\right)$
$\mathrm{Y}^{\prime \prime} \mathrm{z}(-10000)=-5.15 \times 10^{-8}<0-\max$
$Y^{\prime \prime} z(2326.26)=1.55 \times 10^{-8}>0-\min$
$Y^{\prime \prime} z(7673.74)=-2.23 \times 10^{-8}<0-\max$
Calculation of critical points for $x$ axial:
$Y^{\prime} \mathrm{x}=4 \times\left(0.189 \times 10^{-16}\right) \times \mathrm{Kz}^{3}-2 \times\left(0.383 \times 10^{-8}\right) \times \mathrm{Kz}+\left(0.233 \times 10^{-4}\right)=0$
$\frac{1}{10^{-4}} \times\left(7.56 \times 10^{-13} \times \mathrm{Kx}^{3}-7.66 \times 10^{-5} \times \mathrm{Kx}+0.233\right)=0$
The calculation results are as follows:
- $\mathrm{K}_{\mathrm{x} 1}=-11336.2$
- $\mathrm{K}_{\mathrm{x} 2}=3445.45$
- $K_{x 3}=7890.78$
$\mathrm{Y}^{\prime \prime} \mathrm{x}=12 \times\left(0.189 \times 10^{-16}\right) \times \mathrm{Kz}^{2}-2 \times\left(0.383 \times 10^{-8}\right)$
$\mathrm{Y}^{\prime \prime} \mathrm{x}(-10000)=2.15 \times 10^{-8}>0-\min$
$\mathrm{Y}^{\prime \prime} \mathrm{x}(3445.45)=-4.97 \times 10^{-8}<0-\max$
$\mathrm{Y}^{\prime \prime} \mathrm{x}(7890.78)=6.46 \times 10^{-9}<0-\min$
Calculation of critical points for $y$ axial:
$Y^{\prime} y=-4 \times\left(0.204 \times 10^{-16}\right) \times \mathrm{Ky}^{3}+2 \times\left(0.360 \times 10^{-8}\right) \times \mathrm{Ky}-\left(0.171 \times 10^{-4}\right)=0$
$\frac{-1}{10^{-4}} \times\left(8.16 \times 10^{-13} \times \mathrm{Ky}^{3}-7.2 \times 10^{-5} \times \mathrm{Ky}+0.171\right)=0$
The calculation results are as follows:
- $\mathrm{K}_{\mathrm{y} 1}=-10409.9$
- $\mathrm{K}_{\mathrm{y} 2}=2566.62$
- $\mathrm{K}_{\mathrm{y} 3}=7843.28$
$Y^{\prime \prime} y=-12 \times\left(0.204 \times 10^{-16}\right) \times \mathrm{Ky}^{2}+2 \times\left(0.360 \times 10^{-8}\right)$
$Y^{\prime \prime} y(-10409.9)=-1.93 \times 10^{-8}<0-\max$
$\mathrm{Y}^{\prime \prime} \mathrm{y}(2566.62)=4.59 \times 10^{-9}>0-\min$
$\mathrm{Y}^{\prime \prime} \mathrm{x}(78943.28)=-7.86 \times 10^{-9}<0-\max$
Table 30: Summary of Function No.6, Experiment No. 2 with $P=52[w]$

| $\mathbf{K}_{\mathbf{z}}$ | $\mathbf{K}_{\mathbf{x}}$ | $\mathbf{K}_{\mathbf{y}}$ |
| :---: | :---: | :---: |
| 2326.26 | 7890.78 | 2566.62 |

### 3.5. Experiment No. 3

The boundary conditions for Experiment No. 3 are shown in the Table 31 and Figure 35:
Table 31: Boundary Condition of Experiment No. 3

| Thermodynamic parameters | Static Pressure: $101325.00[\mathrm{~Pa}]$ <br> Ambient Temperature: $26\left[{ }^{\circ} \mathrm{C}\right]$ |
| :--- | :--- |
| Material parameters | Default material: Aluminum $6061, \mathrm{~K}=167\left[\frac{\mathrm{~W}}{\mathrm{~m} \times{ }^{\circ} \mathrm{C}}\right]$ <br> Vapor Chamber material: TBD |
| Heat power | $49.92[\mathrm{w}], 82.6[\mathrm{w}]$ or $102.4[\mathrm{w}]$ |
| Contact Resistances | Tflex 220-0.02in(0.51mm)-100psi <br> Faces: Vapor Chamber and Fins Box |



Figure 35: Thermo-Coupler Points of Experiment No. 3
The experiment results are shown in the Table 32:

Table 32: Experiment No. 3 Results

| $\mathrm{I}[\mathrm{amp}]$ | 6.4 | 5.9 | 4.8 |
| :--- | :---: | :---: | :---: |
| $\mathrm{~V}[\mathrm{v}]$ | 16 | 14 | 10.4 |
| $\mathrm{P}[\mathbf{w}]$ | 102.4 | 82.6 | 49.92 |
| Points | $\mathrm{T}\left[{ }^{\circ} \mathrm{C}\right]$ | $\mathrm{T}\left[{ }^{\circ} \mathrm{C}\right]$ | $\mathrm{T}\left[{ }^{\circ} \mathrm{C}\right]$ |
| P 4 | 40.1 | 36.8 | 32.4 |
| P 5 | 30.7 | 29.5 | 28.3 |
| P 6 | 31.5 | 30.1 | 28.7 |
| P 8 | 36.6 | 34.2 | 30.8 |
| P 9 | 35.6 | 33.3 | 30.2 |
| P 10 | 32.6 | 31.0 | 29.1 |
| P 12 | 31.4 | 30.0 | 28.4 |
| P 13 | 27.7 | 27.1 | 26.6 |
| P 14 | 30.5 | 29.2 | 27.9 |

### 3.5.1. Results for Experiment No. 3 with $\mathbf{P}=102.4[w]$

The summary of Experiment No. 3 with $\mathrm{P}=102.4[\mathrm{w}]$ shown in the Table 33:
Table 33: Summary of Experiment No. 3 with $P=102.4[w]$ Results

| $\mathbf{N o}$ | $\boldsymbol{Y}_{\boldsymbol{R M S}}$ | Kz | $\mathbf{K x}$ | $\mathbf{K y}$ |
| :---: | :---: | :---: | :---: | :---: |
| $\mathbf{1}$ | $\mathbf{1 . 4 8 6 3}$ | 500 | 500 | 200 |
| $\mathbf{2}$ | $\mathbf{1 . 4 8 4 8}$ | 500 | 500 | 600 |
| $\mathbf{3}$ | $\mathbf{1 . 4 8 4 5}$ | 500 | 500 | 1000 |
| $\mathbf{4}$ | $\mathbf{1 . 4 8 4 4}$ | 500 | 500 | 1200 |
| $\mathbf{5}$ | $\mathbf{1 . 4 8 4 3}$ | 500 | 500 | 1500 |
| $\mathbf{6}$ | $\mathbf{1 . 4 8 4 2}$ | 500 | 500 | 2000 |
| $\mathbf{7}$ | $\mathbf{1 . 4 8 4 0}$ | 500 | 500 | 3000 |
| $\mathbf{8}$ | $\mathbf{1 . 4 8 4 0}$ | 500 | 500 | 3500 |
| $\mathbf{9}$ | $\mathbf{1 . 4 8 4 0}$ | 500 | 500 | 4000 |
| $\mathbf{1 0}$ | $\mathbf{1 . 4 8 3 6}$ | 500 | 500 | 5000 |
| $\mathbf{1 1}$ | $\mathbf{1 . 4 8 3 6}$ | 500 | 500 | 10000 |
| $\mathbf{1 2}$ | $\mathbf{1 . 4 8 6 7}$ | 600 | 500 | 1200 |
| $\mathbf{1 3}$ | $\mathbf{1 . 5 1 0 5}$ | 700 | 500 | 1200 |
| $\mathbf{1 4}$ | $\mathbf{1 . 5 0 1 3}$ | 800 | 500 | 1200 |


| $\mathbf{N o}$ | $\boldsymbol{Y}_{\text {RMS }}$ | Kz | Kx | Ky |
| :---: | :---: | :---: | :---: | :---: |
| $\mathbf{1 5}$ | $\mathbf{1 . 5 1 0 5}$ | 900 | 500 | 1200 |
| $\mathbf{1 6}$ | $\mathbf{1 . 5 2 0 0}$ | 1000 | 500 | 1200 |
| $\mathbf{1 7}$ | $\mathbf{1 . 5 9 6 7}$ | 2000 | 500 | 1200 |
| $\mathbf{1 8}$ | $\mathbf{1 . 6 2 1 7}$ | 2500 | 500 | 1200 |
| $\mathbf{1 9}$ | $\mathbf{1 . 6 4 0 7}$ | 3000 | 500 | 1200 |
| $\mathbf{2 0}$ | $\mathbf{1 . 6 6 7 4}$ | 4000 | 500 | 1200 |
| $\mathbf{2 1}$ | $\mathbf{1 . 7 2 4 4}$ | 10000 | 500 | 1200 |
| $\mathbf{2 2}$ | $\mathbf{1 . 4 5 7 2}$ | 800 | 400 | 1200 |
| $\mathbf{2 3}$ | $\mathbf{1 . 5 4 1 0}$ | 800 | 600 | 1200 |
| $\mathbf{2 4}$ | $\mathbf{1 . 6 0 7 9}$ | 800 | 800 | 1200 |
| $\mathbf{2 5}$ | $\mathbf{1 . 7 0 4 1}$ | 800 | 1200 | 1200 |
| $\mathbf{2 6}$ | $\mathbf{1 . 7 5 5 1}$ | 800 | 1500 | 1200 |
| $\mathbf{2 7}$ | $\mathbf{1 . 8 1 6 6}$ | 800 | 2000 | 1200 |
| $\mathbf{2 8}$ | $\mathbf{1 . 8 9 2 9}$ | 800 | 3000 | 1200 |
| $\mathbf{2 9}$ | $\mathbf{1 . 9 3 8 7}$ | 800 | 4000 | 1200 |
| $\mathbf{3 0}$ | $\mathbf{1 . 9 6 9 5}$ | 800 | 5000 | 1200 |
| $\mathbf{3 1}$ | $\mathbf{1 . 9 6 9 5}$ | 800 | 10000 | 1200 |

Function No. 7 of Experiment No. 3 with $\mathrm{P}=102.4$ [w]:
$\mathrm{Y}=\mathrm{A} \times \mathrm{Kz}^{3}+\mathrm{B} \times \mathrm{Kx}^{3}+\mathrm{C} \times \mathrm{Ky}^{3}+\mathrm{D} \times \mathrm{Kz}^{2}+\mathrm{E} \times \mathrm{Kx}^{2}+\mathrm{F} \times \mathrm{Ky}^{2}+\mathrm{G} \times \mathrm{Kz}^{5}+\mathrm{H} \times \mathrm{Kx}^{5}+\mathrm{I}$

$$
\times \mathrm{Ky}^{5}+\mathrm{J}
$$

Figure 36 shows that mean parameters values are higher than standard deviation values . It means that the function is right.


Figure 36: Function No. 7 of Experiment No. 3 with $P=102.4$ [w]
The complete Function No. 7 is as follows:

$$
\begin{aligned}
& Y=-\left(0.616 \times 10^{-11}\right) \times \mathrm{Kz}^{3}-\left(0.864 \times 10^{-11}\right) \times \mathrm{Kx}^{3}-\left(0.525 \times 10^{-11}\right) \times \mathrm{Ky}^{3} \\
& +\left(0.345 \times 10^{-7}\right) \times \mathrm{Kz}^{2}+\left(0.597 \times 10^{-7}\right) \times \mathrm{Kx}^{2}+\left(0.194 \times 10^{-7}\right) \times \mathrm{Ky}^{2} \\
& +\left(0.293 \times 10^{-19}\right) \times \mathrm{Kz}^{5}+\left(0.311 \times 10^{-19}\right) \times \mathrm{Kx}^{5}+\left(0.331 \times 10^{-19}\right) \times \mathrm{Ky}^{5}+1.47
\end{aligned}
$$

The Table 34 compares the values received from the Function No. 5 and RMS values.
Table 34: Function Values vs. RMS Values

| $\mathrm{K}_{\mathrm{z}}$ | $\mathrm{K}_{\mathrm{x}}$ | $\mathrm{K}_{\mathrm{y}}$ | Function Value | RMS Value |
| :---: | :---: | :---: | :---: | :---: |
| 2000 | 500 | 1200 | 1.59245 | 1.52 |

Calculation of critical points for $z$ axial:

$$
\begin{aligned}
\mathrm{Y}^{\prime} \mathrm{z}=-3 \times( & \left.0.616 \times 10^{-11}\right) \times \mathrm{Kz}^{2}+2 \times\left(0.345 \times 10^{-7}\right) \times \mathrm{Kz}+5 \times\left(0.293 \times 10^{-19}\right) \times \mathrm{Kz}^{4} \\
& =0
\end{aligned}
$$

$\frac{\mathrm{Kz}}{10^{-7}} \times\left(\left(1.465 \times 10^{-12}\right) \times \mathrm{Kz}^{3}-\left(1.848 \times 10^{-4}\right) \times \mathrm{Kz}+0.69\right)=0$
The calculation results are as follows:

- $\mathrm{K}_{\mathrm{z} 1}=-12768.3$
- $\mathrm{K}_{\mathrm{z} 2}=4416.85$
- $K_{z 3}=8351.5$
$\mathrm{Y}^{\prime \prime} \mathrm{z}=20 \times\left(0.293 \times 10^{-19}\right) \times \mathrm{Kz}^{3}-6 \times\left(0.616 \times 10^{-11}\right) \times \mathrm{Kz}+2 \times\left(0.345 \times 10^{-7}\right)$
$\mathrm{Y}^{\prime \prime} \mathrm{z}(4416.85)=-4.4 \times 10^{-7}<0-\max$.
$\mathrm{Y}^{\prime \prime} \mathrm{z}(8351.5)=1.02 \times 10^{-6}>0-\min$.
Calculation of critical points for $x$ axial:
$\mathrm{Y}^{\prime} x=-3 \times\left(0.864 \times 10^{-11}\right) \times \mathrm{Kx}^{2}+2 \times\left(0.597 \times 10^{-7}\right) \times \mathrm{Kx}+5 \times\left(0.311 \times 10^{-19}\right)$

$$
\times K x^{4}=0
$$

$\frac{\mathrm{Kx}}{10^{-7}} \times\left(\left(1.555 \times 10^{-12}\right) \times \mathrm{Kx}^{3}-\left(2.592 \times 10^{-4}\right) \times \mathrm{Kx}+1.194\right)=0$
The calculation results are as follows:

- $\mathrm{K}_{\mathrm{x} 1}=-14785.8$
- $\mathrm{K}_{\mathrm{x} 2}=5742.58$
- $\mathrm{K}_{\mathrm{x} 3}=9043.2$
$\mathrm{Y}^{\prime \prime} x=20 \times\left(0.311 \times 10^{-19}\right) \times \mathrm{Kx}^{3}-6 \times\left(0.864 \times 10^{-11}\right) \times \mathrm{Kx}+2 \times\left(0.597 \times 10^{-7}\right)$
$\mathrm{Y}^{\prime} x(5742.58)=-6.05 \times 10^{-7}<0-\max$.
$\mathrm{Y}^{\prime \prime} x(9043.2)=1.11 \times 10^{-6}>0-\min$.
Calculation of critical points for $y$ axial:
$\mathrm{Y}^{\prime} y=-3 \times\left(0.525 \times 10^{-11}\right) \times \mathrm{Ky}^{2}+2 \times\left(0.194 \times 10^{-7}\right) \times \mathrm{Ky}+5 \times\left(0.331 \times 10^{-19}\right)$

$$
\times K y^{4}=0
$$

$\frac{\mathrm{Ky}}{10^{7}} \times\left(\left(1.655 \times 10^{-12}\right) \times \mathrm{Ky}^{3}-\left(1.575 \times 10^{-4}\right) \times \mathrm{Ky}+0.388\right)=0$
The calculation results are as follows:

- $\mathrm{K}_{\mathrm{y} 1}=-10809.9$
- $\mathrm{K}_{\mathrm{y} 2}=2661.63$
- $\mathrm{K}_{\mathrm{y} 3}=8148.27$
$\mathrm{Y}^{\prime \prime} y=20 \times\left(0.331 \times 10^{-19}\right) \times \mathrm{Ky}^{3}-6 \times\left(0.525 \times 10^{-11}\right) \times \mathrm{Ky}+2 \times\left(0.194 \times 10^{-7}\right)$
$\mathrm{Y}^{\prime \prime} y(2661.63)=-3.26 \times 10^{-7}<0-\max$.
$\mathrm{Y}^{\prime \prime} y(8148.27)=1.4 \times 10^{-6}>0-\min$.
Table 35: Minimum of Function No.7, Experiment No. 3 with $P=102.4[w]$

| $\mathbf{K}_{\mathbf{z}}$ | $\mathbf{K}_{\mathbf{x}}$ | $\mathbf{K}_{\mathbf{y}}$ |
| :---: | :---: | :---: |
| 8351.5 | 9043.2 | 8148.27 |

### 3.5.2. Results for Experiment No. 3 with $\mathbf{P}=\mathbf{8 2 . 6}[\mathbf{w}]$

The summary of Experiment No. 3 with $\mathrm{P}=82.6[\mathrm{w}]$ shown in the Table 36 :
Table 36: Summary of Experiment No. 3 with $\mathbf{P}=\mathbf{8 2 . 6 [ w ]}$ Results

| No | $Y_{\text {RMS }}$ | Kz | Kx | Ky |
| :---: | :---: | :---: | :---: | :---: |
| 1 | 1.3112 | 500 | 500 | 200 |
| 2 | 1.3063 | 500 | 500 | 600 |
| 3 | 1.3053 | 500 | 500 | 1000 |
| 4 | 1.3051 | 500 | 500 | 1200 |
| 5 | 1.3048 | 500 | 500 | 1500 |
| 6 | 1.3046 | 500 | 500 | 2000 |
| 7 | 1.3043 | 500 | 500 | 3000 |
| 8 | 1.3042 | 500 | 500 | 3500 |
| 9 | 1.3041 | 500 | 500 | 4000 |
| 10 | 1.3039 | 500 | 500 | 5000 |
| 11 | 1.3036 | 500 | 500 | 10000 |
| 12 | 1.3062 | 600 | 500 | 1200 |
| 13 | 1.3104 | 700 | 500 | 1200 |
| 14 | 1.3160 | 800 | 500 | 1200 |
| 15 | 1.3225 | 900 | 500 | 1200 |
| 16 | 1.3292 | 1000 | 500 | 1200 |
| 17 | 1.3359 | 1100 | 500 | 1200 |
| 18 | 1.3160 | 1200 | 500 | 1200 |
| 19 | 1.3489 | 1300 | 500 | 1200 |
| 20 | 1.3607 | 1500 | 500 | 1200 |
| 21 | 1.3854 | 2000 | 500 | 1200 |
| 22 | 1.4376 | 4000 | 500 | 1200 |
| 23 | 1.4509 | 5000 | 500 | 1200 |
| 24 | 1.3165 | 1200 | 400 | 1200 |
| 25 | 1.3664 | 1200 | 600 | 1200 |
| 26 | 1.4074 | 1200 | 800 | 1200 |


| No | $\boldsymbol{Y}_{\boldsymbol{R M S}}$ | Kz | Kx | Ky |
| :---: | :---: | :---: | :---: | :---: |
| $\mathbf{2 7}$ | $\mathbf{1 . 4 4 0 4}$ | 1200 | 1000 | 1200 |
| $\mathbf{2 8}$ | $\mathbf{1 . 4 9 9 9}$ | 1200 | 1500 | 1200 |
| $\mathbf{2 9}$ | $\mathbf{1 . 5 3 9 5}$ | 1200 | 2000 | 1200 |
| $\mathbf{3 0}$ | $\mathbf{1 . 5 8 9 3}$ | 1200 | 3000 | 1200 |
| $\mathbf{3 1}$ | $\mathbf{1 . 6 1 9 6}$ | 1200 | 4000 | 1200 |
| $\mathbf{3 2}$ | $\mathbf{1 . 6 4 0 4}$ | 1200 | 5000 | 1200 |
| $\mathbf{3 3}$ | $\mathbf{1 . 6 8 9 7}$ | 1200 | 10000 | 1200 |

Function No. 8 of Experiment No. 3 with $\mathrm{P}=82.6$ [w]:

$$
\begin{aligned}
Y=A \times K z^{2} & +B \times K x^{2}+C \times K y^{2}+D \times K z^{3}+E \times K x^{3}+F \times K y^{3}+G \times K z^{5}+H \times K x^{5}+I \\
& \times K y^{5}+J
\end{aligned}
$$

Figure 37 shows that mean parameters values are higher than standard deviation values. It means that the function is right.


Figure 37: Function No. 8 of Experiment No. 3 with $\mathrm{P}=82.6$ [w]
The complete Function No. 8 is as follows:
$\mathrm{Y}=\left(0.351 \times 10^{-7}\right) \times \mathrm{Kz}^{2}+\left(0.435 \times 10^{-7}\right) \times \mathrm{Kx}^{2}+\left(0.176 \times 10^{-7}\right) \times \mathrm{Ky}^{2}$
$-\left(0.653 \times 10^{-11}\right) \times \mathrm{Kz}^{3}-\left(0.665 \times 10^{-11}\right) \times \mathrm{Kx}^{3}-\left(0.461 \times 10^{-11}\right) \times \mathrm{Ky}^{3}$
$+\left(0.205 \times 10^{-19}\right) \times \mathrm{Kz}^{5}+\left(0.265 \times 10^{-19}\right) \times \mathrm{Kx}^{5}+\left(0.285 \times 10^{-19}\right) \times \mathrm{Ky}^{5}+1.28$
The Table 37 compares the values received from the Function No. 5 and RMS values.

Table 37: Function Values vs. RMS Values

| $\mathrm{K}_{\mathrm{z}}$ | $\mathrm{K}_{\mathrm{x}}$ | $\mathrm{K}_{\mathrm{y}}$ | Function Value | RMS Value |
| :---: | :---: | :---: | :---: | :---: |
| 1200 | 2000 | 1200 | 1.462104 | 1.5395 |

Calculation of critical points for $z$ axial:
$Y^{\prime} z=2 \times\left(0.351 \times 10^{-7}\right) \times K z-3 \times\left(0.653 \times 10^{-11}\right) \times \mathrm{Kz}^{2}+5 \times\left(0.205 \times 10^{-19}\right) \times \mathrm{Kz}^{4}$

$$
=0
$$

$\frac{\mathrm{Kz}}{10^{-7}} \times\left(\left(1.025 \times 10^{-12}\right) \times \mathrm{Kz}^{3}-\left(1.959 \times 10^{-4}\right) \times \mathrm{Kz}+0.702\right)=0$
The calculation results are as follows:

- $\mathrm{K}_{\mathrm{z} 1}=-15353.5$
- $\mathrm{K}_{\mathrm{z} 2}=3891.9$
- $\mathrm{K}_{\mathrm{z} 3}=8396.3$
$\mathrm{Y}^{\prime \prime} \mathrm{z}=20 \times\left(0.205 \times 10^{-19}\right) \times \mathrm{Kz}^{3}-6 \times\left(0.653 \times 10^{-11}\right) \times \mathrm{Kz}+2 \times\left(0.351 \times 10^{-7}\right)$
$\mathrm{Y}^{\prime \prime} \mathrm{z}(3891.9)=-5.81 \times 10^{-7}<0-\max$.
$\mathrm{Y}^{\prime \prime} \mathrm{z}(8396.3)=3.77 \times 10^{-7}>0-$ min.
Calculation of critical points for $x$ axial:
$\mathrm{Y}^{\prime} x=2 \times\left(0.435 \times 10^{-7}\right) \times \mathrm{Kx}-3 \times\left(0.665 \times 10^{-11}\right) \times \mathrm{Kx}^{2}+5 \times\left(0.265 \times 10^{-19}\right) \times \mathrm{Kx}^{4}$

$$
=0
$$

$\frac{\mathrm{Kx}}{10^{-7}} \times\left(\left(1.325 \times 10^{-12}\right) \times \mathrm{Kx}^{3}-\left(1.995 \times 10^{-4}\right) \times \mathrm{Kx}+0.870\right)=0$
The calculation results are as follows:

- $\mathrm{K}_{\mathrm{x} 1}=-14046.7$
- $\mathrm{K}_{\mathrm{x} 2}=5416.1$
- $K_{\mathrm{x} 3}=8630.62$
$\mathrm{Y}^{\prime \prime} x=20 \times\left(0.265 \times 10^{-19}\right) \times \mathrm{Kx}^{3}-6 \times\left(0.665 \times 10^{-11}\right) \times \mathrm{Kx}+2 \times\left(0.435 \times 10^{-7}\right)$
$\mathrm{Y}^{\prime \prime} x(5415.1)=-4.5 \times 10^{-7}<0-\max$.
$\mathrm{Y}^{\prime \prime} x(8630.62)=8.3 \times 10^{-7}>0-\min$.
Calculation of critical points for $y$ axial:
$\mathrm{Y}^{\prime} y=2 \times\left(0.176 \times 10^{-7}\right) \times \mathrm{Ky}-3 \times\left(0.461 \times 10^{-11}\right) \times \mathrm{Ky}^{2}+5 \times\left(0.285 \times 10^{-19}\right) \times \mathrm{Ky}^{4}$

$$
=0
$$

$\frac{\mathrm{Ky}}{10^{-7}} \times\left(\left(1.425 \times 10^{-12}\right) \times \mathrm{Ky}^{3}-\left(1.383 \times 10^{-4}\right) \times \mathrm{Ky}+0.352\right)=0$
The calculation results are as follows:

- $\mathrm{K}_{\mathrm{y} 1}=-10937.8$
- $\mathrm{K}_{\mathrm{y} 2}=2762.38$
- $\mathrm{K}_{\mathrm{y} 3}=8175.46$
$\mathrm{Y}^{\prime \prime} y=20 \times\left(0.285 \times 10^{-19}\right) \times \mathrm{Ky}^{3}-6 \times\left(0.461 \times 10^{-11}\right) \times \mathrm{Ky}+2 \times\left(0.176 \times 10^{-7}\right)$
$\mathrm{Y}^{\prime \prime} y(2762.38)=-2.9 \times 10^{-7}<0-\max$.
$\mathrm{Y}^{\prime \prime} y(8175.46)=1.21 \times 10^{-6}>0-\min$.
Table 38: Minimum of Function No.8, Experiment No. 3 with $P$ with $P=82.6[w]$

| $\mathbf{K}_{\mathbf{z}}$ | $\mathbf{K}_{\mathbf{x}}$ | $\mathbf{K}_{\mathbf{y}}$ |
| :---: | :---: | :---: |
| 8396.3 | 8630.62 | 8175.46 |

### 3.5.3. Results for Experiment No. 3 with $P=49.92[w]$

The summary of Experiment No. 3 with $\mathrm{P}=49.92$ [w] shown in the Table 39:
Table 39: Summary of Experiment No. 3 with $\mathrm{P}=49.92$ [w] Results

| No | $\boldsymbol{Y}_{\boldsymbol{R M S}}$ | Kz | $\mathbf{K x}$ | Ky |
| :---: | :---: | :---: | :---: | :---: |
| $\mathbf{1}$ | $\mathbf{0 . 7 5 6 6}$ | 500 | 500 | 200 |
| $\mathbf{2}$ | $\mathbf{0 . 7 5 3 6}$ | 500 | 500 | 600 |
| $\mathbf{3}$ | $\mathbf{0 . 7 5 3 0}$ | 500 | 500 | 1000 |
| $\mathbf{4}$ | $\mathbf{0 . 7 5 2 8}$ | 500 | 500 | 1200 |
| $\mathbf{5}$ | $\mathbf{0 . 7 5 2 6}$ | 500 | 500 | 1500 |
| $\mathbf{6}$ | $\mathbf{0 . 7 5 2 5}$ | 500 | 500 | 2000 |
| $\mathbf{7}$ | $\mathbf{0 . 7 5 2 3}$ | 500 | 500 | 3000 |
| $\mathbf{8}$ | $\mathbf{0 . 7 5 2 2}$ | 500 | 500 | 3500 |
| $\mathbf{9}$ | $\mathbf{0 . 7 5 2 1}$ | 500 | 500 | 4000 |


| $\mathbf{N o}$ | $\boldsymbol{Y}_{\text {RMS }}$ | Kz | Kx | Ky |
| :---: | :---: | :---: | :---: | :---: |
| $\mathbf{1 0}$ | $\mathbf{0 . 7 5 2 0}$ | 500 | 500 | 5000 |
| $\mathbf{1 1}$ | $\mathbf{0 . 7 5 1 8}$ | 500 | 500 | 10000 |
| $\mathbf{1 2}$ | $\mathbf{0 . 7 5 1 8}$ | 600 | 500 | 1200 |
| $\mathbf{1 3}$ | $\mathbf{0 . 7 5 2 8}$ | 700 | 500 | 1200 |
| $\mathbf{1 4}$ | $\mathbf{0 . 7 5 5 1}$ | 800 | 500 | 1200 |
| $\mathbf{1 5}$ | $\mathbf{0 . 7 5 7 9}$ | 900 | 500 | 1200 |
| $\mathbf{1 6}$ | $\mathbf{0 . 7 6 1 1}$ | 1000 | 500 | 1200 |
| $\mathbf{1 7}$ | $\mathbf{0 . 7 8 9 7}$ | 1200 | 500 | 1200 |
| 18 | $\mathbf{0 . 7 8 9 9}$ | 2000 | 500 | 1200 |
| 19 | $\mathbf{0 . 8 0 7 3}$ | 3000 | 500 | 1200 |
| 20 | $\mathbf{0 . 8 1 8 2}$ | 4000 | 500 | 1200 |
| $\mathbf{2 1}$ | $\mathbf{0 . 8 4 1 4}$ | 10000 | 500 | 1200 |
| $\mathbf{2 2}$ | $\mathbf{0 . 7 5 5 0}$ | 1200 | 400 | 1200 |
| $\mathbf{2 3}$ | $\mathbf{0 . 7 7 9 7}$ | 1200 | 600 | 1200 |
| $\mathbf{2 4}$ | $\mathbf{0 . 8 0 1 3}$ | 1200 | 800 | 1200 |
| $\mathbf{2 5}$ | $\mathbf{0 . 8 1 9 3}$ | 1200 | 1000 | 1200 |
| $\mathbf{2 6}$ | $\mathbf{0 . 8 5 2 3}$ | 1200 | 1500 | 1200 |
| $\mathbf{2 7}$ | $\mathbf{0 . 8 7 8 9}$ | 1200 | 2000 | 1200 |
| $\mathbf{2 8}$ | $\mathbf{0 . 9 0 3 0}$ | 1200 | 3000 | 1200 |
| $\mathbf{2 9}$ | $\mathbf{0 . 9 2 0 5}$ | 1200 | 4000 | 1200 |
| $\mathbf{3 0}$ | $\mathbf{0 . 9 3 2 5}$ | 1200 | 5000 | 1200 |
| $\mathbf{3 1}$ | $\mathbf{0 . 9 6 1 1}$ | 1200 | 10000 | 1200 |
|  |  |  |  |  |
| $\mathbf{y y}$ |  |  |  |  |

Function No. 9 of Experiment No. 3 with $\mathrm{P}=49.92$ [w]:

$$
\begin{aligned}
Y=A \times K z^{2}+ & B \times K x^{2}+C \times K y^{2}+D \times K z^{3}+E \times K x^{3}+F \times K y^{3}+G \times K z^{5}+H \times K x^{5}+I \\
& \times K y^{5}+J
\end{aligned}
$$

Figure 38 shows that mean parameters values are higher than standard deviation values. It means that the function is right.


Figure 38: Function No. 9 of Experiment No. 3 with $\mathrm{P}=49.92$ [w]
The complete Function No. 9 is as follows:
$Y=\left(0.143 \times 10^{-7}\right) \times K z^{2}+\left(0.240 \times 10^{-7}\right) \times K x^{2}+\left(0.982 \times 10^{-8}\right) \times \mathrm{Ky}^{2}$
$-\left(0.276 \times 10^{-11}\right) \times \mathrm{Kz}^{3}-\left(0.370 \times 10^{-11}\right) \times \mathrm{Kx}^{3}-\left(0.260 \times 10^{-11}\right) \times \mathrm{Ky}^{3}$
$+\left(0.142 \times 10^{-19}\right) \times \mathrm{Kz}^{5}+\left(0.150 \times 10^{-19}\right) \times \mathrm{Kx}^{5}+\left(0.162 \times 10^{-19}\right) \times \mathrm{Ky}^{5}+0.743$
The Table 40 compares the values received from the Function No. 5 and RMS values.
Table 40: Function Values vs. RMS Values

| $\mathrm{K}_{\mathrm{z}}$ | $\mathrm{K}_{\mathrm{x}}$ | $\mathrm{K}_{\mathrm{y}}$ | Function Value | RMS Value |
| :---: | :---: | :---: | :---: | :---: |
| 1200 | 2000 | 1200 | 0.835 | 0.8789 |

Calculation of critical points for $z$ axial:
$Y^{\prime} z=2 \times\left(0.143 \times 10^{-7}\right) \times \mathrm{Kz}-3 \times\left(0.276 \times 10^{-11}\right) \times \mathrm{Kz}^{2}+5 \times\left(0.142 \times 10^{-19}\right) \times \mathrm{Kz}^{4}$
$=0$
$\frac{\mathrm{Kz}}{10^{-7}} \times\left(\left(0.71 \times 10^{-12}\right) \times \mathrm{Kz}^{3}-\left(0.828 \times 10^{-4}\right) \times \mathrm{Kz}+0.286\right)=0$
The calculation results are as follows:

- $\mathrm{K}_{\mathrm{z} 1}=-12229.4$
- $\mathrm{K}_{\mathrm{z} 2}=4004.93$
- $K_{\mathrm{z} 3}=8224.47$
$\mathrm{Y}^{\prime \prime} \mathrm{z}=20 \times\left(0.142 \times 10^{-19}\right) \times \mathrm{Kz}^{3}-6 \times\left(0.276 \times 10^{-11}\right) \times \mathrm{Kz}^{2}+2 \times\left(0.143 \times 10^{-7}\right) \times \mathrm{Kz}$ $\mathrm{Y}^{\prime \prime} \mathrm{z}(4004.93)=-1.95 \times 10^{-7}<0-\max$.
$\mathrm{Y}^{\prime \prime} \mathrm{z}(8224.47)=5.04 \times 10^{-7}>0-\min$.
Calculation of critical points for $x$ axial:
$\mathrm{Y}^{\prime} x=2 \times\left(0.240 \times 10^{-7}\right) \times \mathrm{Kx}-3 \times\left(0.370 \times 10^{-11}\right) \times \mathrm{Kx}^{2}+5 \times\left(0.150 \times 10^{-19}\right) \times \mathrm{Kx}^{4}$ $=0$
$\frac{\mathrm{Kx}}{10^{-7}} \times\left(\left(0.75 \times 10^{-12}\right) \times \mathrm{Kx}^{3}-\left(1.11 \times 10^{-4}\right) \times \mathrm{Kx}+0.48\right)=0$
The calculation results are as follows:
- $\mathrm{K}_{\mathrm{x} 1}=-13926.8$
- $\mathrm{K}_{\mathrm{x} 2}=5371.53$
- $K_{\mathrm{x} 3}=8555.23$
$\mathrm{Y}^{\prime \prime} x=20 \times\left(0.150 \times 10^{-19}\right) \times \mathrm{Kx}^{3}-6 \times\left(0.370 \times 10^{-11}\right) \times \mathrm{Kx}+2 \times\left(0.240 \times 10^{-7}\right)$
$\mathrm{Y}^{\prime \prime} x(5371.53)=-2.5 \times 10^{-7}<0-\max$.
$\mathrm{Y}^{\prime \prime} x(8555.23)=4.6 \times 10^{-7}>0-\min$.
Calculation of critical points for $y$ axial:
$\mathrm{Y}^{\prime} y=2 \times\left(0.982 \times 10^{-8}\right) \times \mathrm{Ky}-3 \times\left(0.260 \times 10^{-11}\right) \times \mathrm{Ky}^{2}+5 \times\left(0.162 \times 10^{-19}\right) \times \mathrm{Ky}^{4}$

$$
=0
$$

$\frac{\mathrm{Ky}}{10^{-8}} \times\left(\left(0.81 \times 10^{-11}\right) \times \mathrm{Ky}^{3}-\left(0.78 \times 10^{-3}\right) \times \mathrm{Ky}+1.964\right)=0$
The calculation results are as follows:

- $\mathrm{K}_{\mathrm{y} 1}=-10888.7$
- $\mathrm{K}_{\mathrm{y} 2}=2729.01$
- $\mathrm{K}_{\mathrm{y} 3}=8159.71$
$\mathrm{Y}^{\prime \prime} y=20 \times\left(0.162 \times 10^{-19}\right) \times \mathrm{Ky}^{3}-6 \times\left(0.260 \times 10^{-11}\right) \times \mathrm{Ky}+2 \times\left(0.982 \times 10^{-8}\right)$
$\mathrm{Y}^{\prime \prime} y(2729.01)=-1.6 \times 10^{-7}<0-\max$.
$\mathrm{Y}^{\prime \prime} y(8159.71)=2.45 \times 10^{-6}>0-\min$.


### 3.6. All-Inclusive Results Summary

The Table 33 shows minimum values received during the experiments described within this project. These set of values constitute an effective thermal conductivity of Vapor Chamber.

Table 41: Summary of Effective Thermal Conductivities

| $\mathbf{K}_{\mathbf{i}}$ <br> Experiment | $\mathbf{K}_{\mathbf{z}}$ | $\mathbf{K}_{\mathbf{x}}$ | $\mathbf{K}_{\mathbf{y}}$ |
| :---: | :---: | :---: | :---: |
| $\mathbf{1 - 1 1 4 [ \mathbf { w } ]}$ | 8322.86 | 8395.05 | 8316 |
| $\mathbf{1 - 5 7}[\mathbf{w}]$ | 8206.58 | 8250.21 | 8266.64 |
| $\mathbf{2 - 1 1 0 . 7 2 [ w ] ~}$ | 12122 | 7563.56 | 12872.5 |
| $\mathbf{2 - 8 0 . 9 4}[\mathbf{w}]$ | 16647.9 | 12240.8 | 7656.55 |
| $\mathbf{2 - 5 2 [ w ] ~}$ | 2326.26 | 7890.78 | 2566.62 |
| $\mathbf{3 - 1 0 2 . 4}[\mathbf{w}]$ | 8351.5 | 9043.2 | 8148.27 |
| $\mathbf{3 - 8 2 . 6}[\mathbf{w}]$ | 8396.3 | 8630.65 | 8175.46 |
| $\mathbf{3 - 4 9 . 9 4}[\mathbf{w}]$ | 8224.47 | 8555.23 | 8159.71 |

## 4. Conclusions

- The experiments performed within the framework of this project have shown that the Vapor Chamber is one of the effective thermally spread component in electronic cooling system with thermal conductivities more than 20 time higher than copper.
- Based on experiments $1 \& 3$
> Vapor Chamber functions as isotropic body with high thermal conductivity
$>$ Properties of a Vapor Chamber do not depend on the power changes
- Based on experiment 2
> Fluid and Vapor flows inside the Vapor Chamber component become unstable, if the heat source is located at a place, where the channels are branched
- It is not recommended to put the TEC on the location described in the Experiment 2 due to unpredictable performance of the Vapor Chamber at this configuration
- The defined properties of the Vapor Chamber can be used for thermal design of cooling system based on combination of Thermoelectric and Vapor Chamber technologies
- Although the Vapor Chamber component is widely used in electronic cooling technologies, to discover all thermal properties of the Vapor Chamber still requires a deep research.


## 5. References

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